Oddtown, Eventown

by Károly Zsolnai

There is a town with 100 people who all love forming clubs. There are clubs for poker and chess players, golfers, divorced women, you name it. One day the mayor decides that the town is overcrowded with clubs, and it can't go on any further: restrictions should be given to their numbers, and therefore imposes two rules. The first rule is that **a club may only exist if it has an *odd* number of members**, and the second rule is that **any two clubs may only have an even number of common members**.

The question is: how many clubs may be formed from now on? What is the maximum?

The answer is: **if there are 100 citizens, if they do it right, they may form exactly 100 clubs.**

Simple enough. So, what's the point? Well, let's turn the story to another way. What if we're playing Eventown instead of Oddtown, and impose the following rules: **a club may only exist if it has an *even* number of members**, and the second rule is that **any two clubs may only have an even number of common members**.

It's basically the same, only the first rule have been changed from *odd* to *even*. How many clubs may be formed now? Since the question is basically the same (or almost the same), why do we even bother? The answer has to be maybe one less or one more, but probably the same.

The answer is: $\mathbf{1,125,899,906,842,624}$.

One quadrillion, 125 trillion, 899 billion, 906 million, 842 thousands and 624. According to Wolfram Alpha, it's roughly the number of red cells in the human body. Well, almost got it, right?

Mathematically speaking, an Oddtown of n inhabitants can form up to n clubs, and Eventown may form $2^{\lfloor \frac{n}{2} \rfloor}$.

The conclusion? Mathematics is so divine, it has so great depths, while our mind is very earth-bound, very limited, often stuck with common sense and taking simple but false assumptions for granted. This is indeed a very important teaching.