Scalar fields, vector fields, divergence, curl

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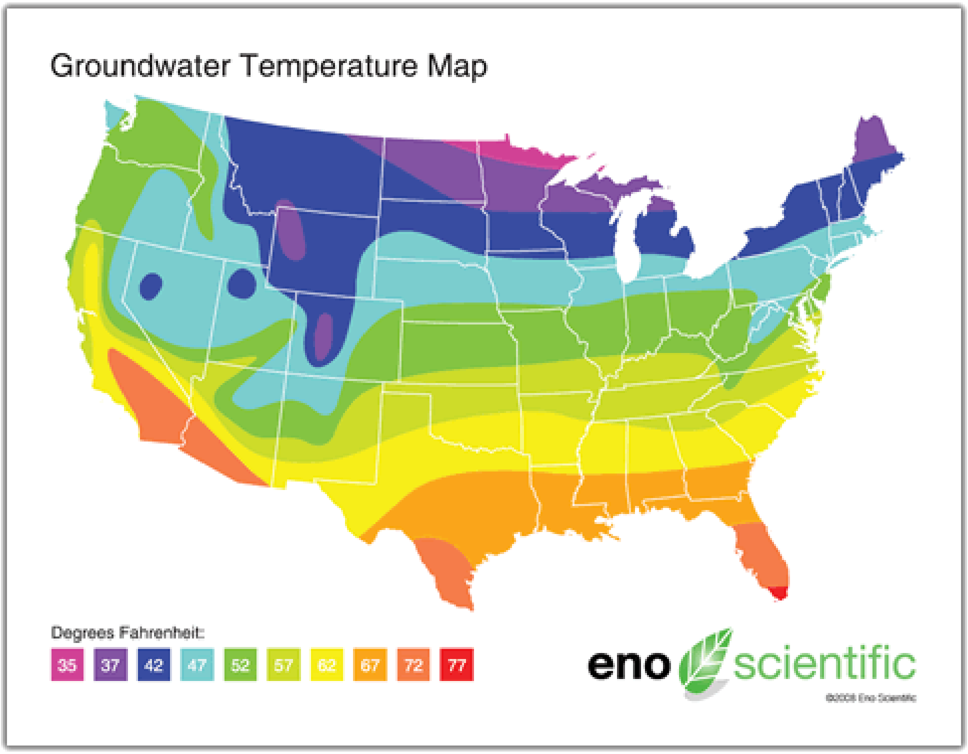
These complicated names bear quite simple meanings. Let's see!

Scalar fields

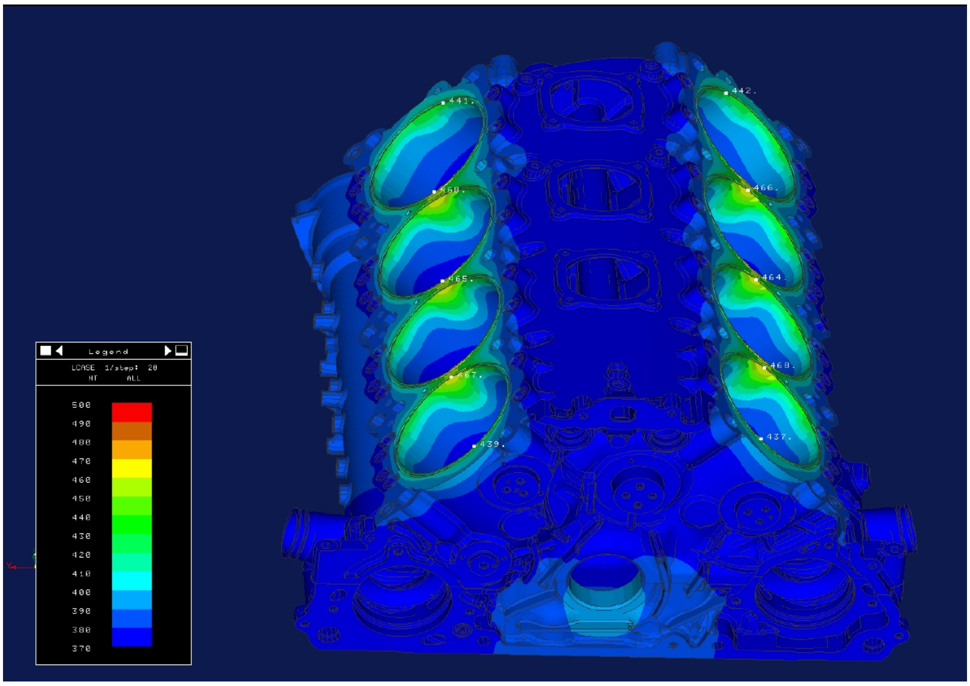
Consider any kind two dimensional space. For instance, the map of the United States, and for every point in this map, we measure and indicate the groundwater temperature.



1. Figure. The temperature map of the USA.

You may consider the room you're sitting in (it is a three dimensional space), where you can also tell at which point what the temperature is. If you're an engineer, you may also be curious about the temperature distribution in an engine you're designing (figure 3). We have one arbitrary dimension space, and to every point in this space we associate a number. This number can mean anything: the average temperatures or average salaries on a map, or maybe the pressure of a fluid. *This is it, these are what mathematicians call scalar fields.*

After a pleasant winter time walk, you enter your house, and you feel that it is quite cold in there. As you'd like to warm up as soon as possible, where would



2. Figure. A three dimensional temperature map

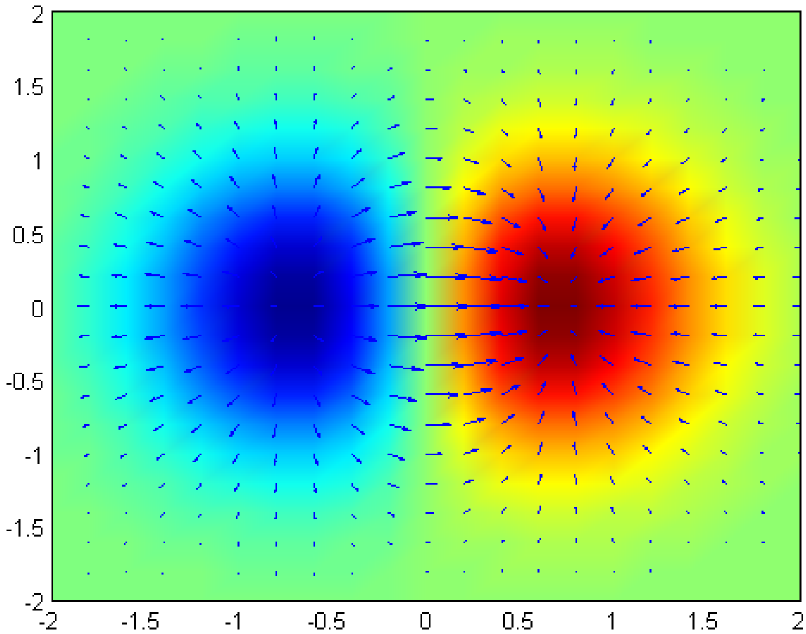
you move? Exactly in the direction where you experience the biggest increase in temperature. This direction, for every point is defined by the **gradient of the scalar field**. The **gradient** of this temperature map will always point from the colder parts to the parts with the highest temperature increase (figure 1). Figure 3 is also a great visualization of the gradient vectors[1].

Vector fields

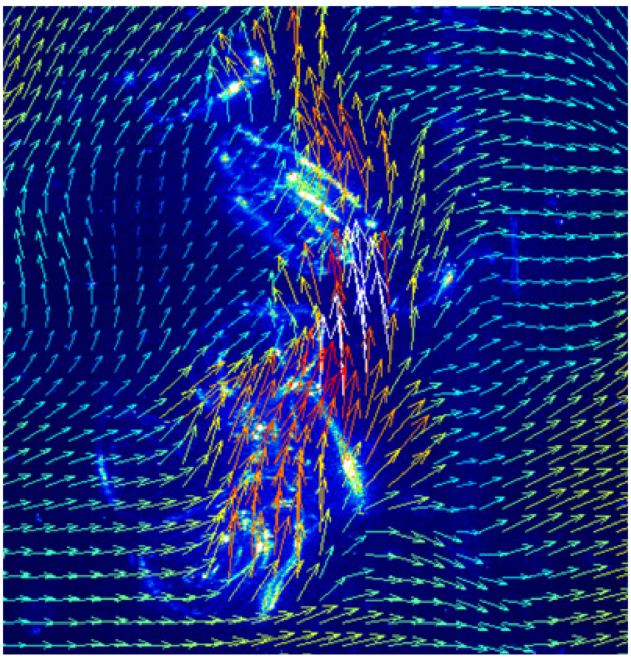
These are the same as scalar fields, but to the points in space, *not scalar values (numbers), but vectors are assigned*. As the wind the does not only have strength, but a direction, a wind map is a good example for this. A fluid velocity field is a field where for every point in space, the velocity is given, which indicates the direction the fluid is flowing and where it will flow the next moment.

This video is a great demonstration of a fluid velocity field changing in time. As forces have directions, a force field would make a velocity field as well.

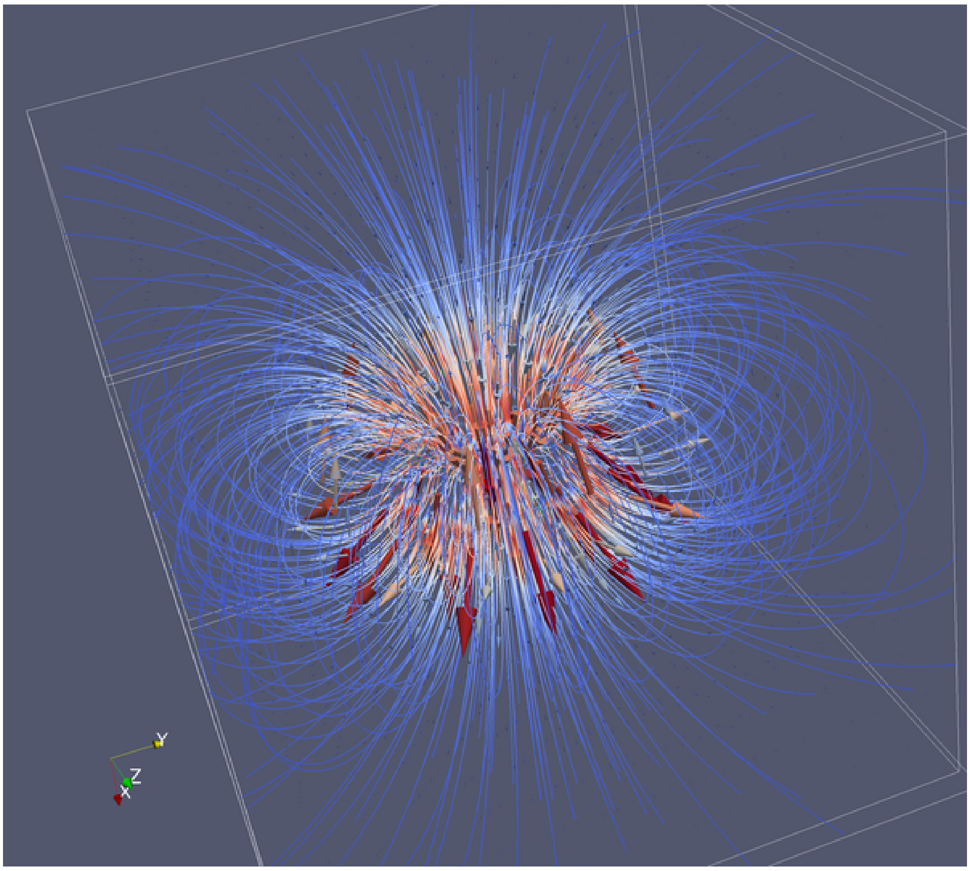
[1]source: `http://en.wikipedia.org/wiki/Gradient`



3. Figure. A great visualization of the gradients for the points in this temperature (or height) map.



4. Figure. The velocity field of a fluid, indicating the direction of the flow.



5. Figure. Visualization of a 3D vector field.

Divergence

Draw any kind of closed shape in the fluid velocity field (figure 4). It can be a box, or anything you wish, as soon at this surface is closed. Watch the fluid in and around this shape in time: if more fluid flows in this shape than it flows outwards, we say that the **divergence of the velocity in that part of the fluid is positive**. If so, the change of the velocity at some parts has to be higher than others. This can't happen without any exterior help: velocity does not increase just by itself, so *there must be a source* that is responsible for this change. If more fluid flows out of the shape than comes in, we have a sink somewhere which is responsible for the disappeared velocity, and the **divergence of the velocity is negative** there. **Zero divergence** means there aren't any of sinks or sources in that part of the fluid.

It's also a good example that if you perceive that smoke comes from a particular direction to you, there has to be someone who is smoking (he or she is the source). If the smoke disappears somewhere, you can very well assume that there is someone inhaling it (sink).

Maxwell made observations about the sources and sinks of magnetic and electric fields in his famous equations.

Curl

I'm sure you have seen the Niagara waterfall, at least on picture. What would happen if you could put a ball in the way of the water? Of course, the water would start to *rotate* it. Consider the fluid velocity field (figure 4): **if you would put a little ball in a chosen point, where would it start to rotate**? If it wouldn't, the **curl** (rotation) is zero at that particular point, otherwise it will show the direction and the angular speed of the rotation (how fast it makes the ball turn in that particular direction).