Teil 3: über Aliasing

Sampling, Rekonstruktion

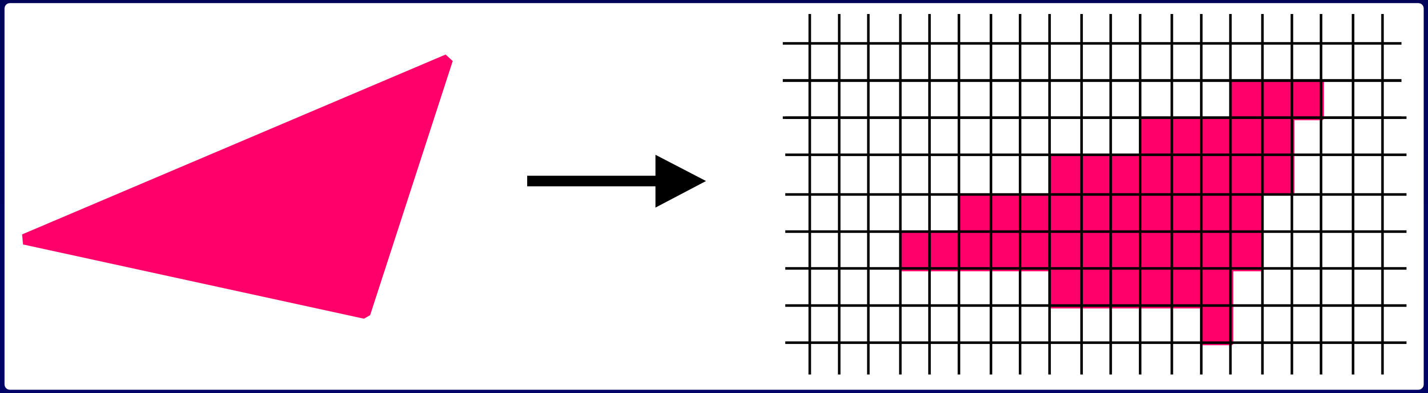
Motivation

Aliasing – was ist das?

- Probleme aus Ungleichung

  kontinuierlich ≠ diskret

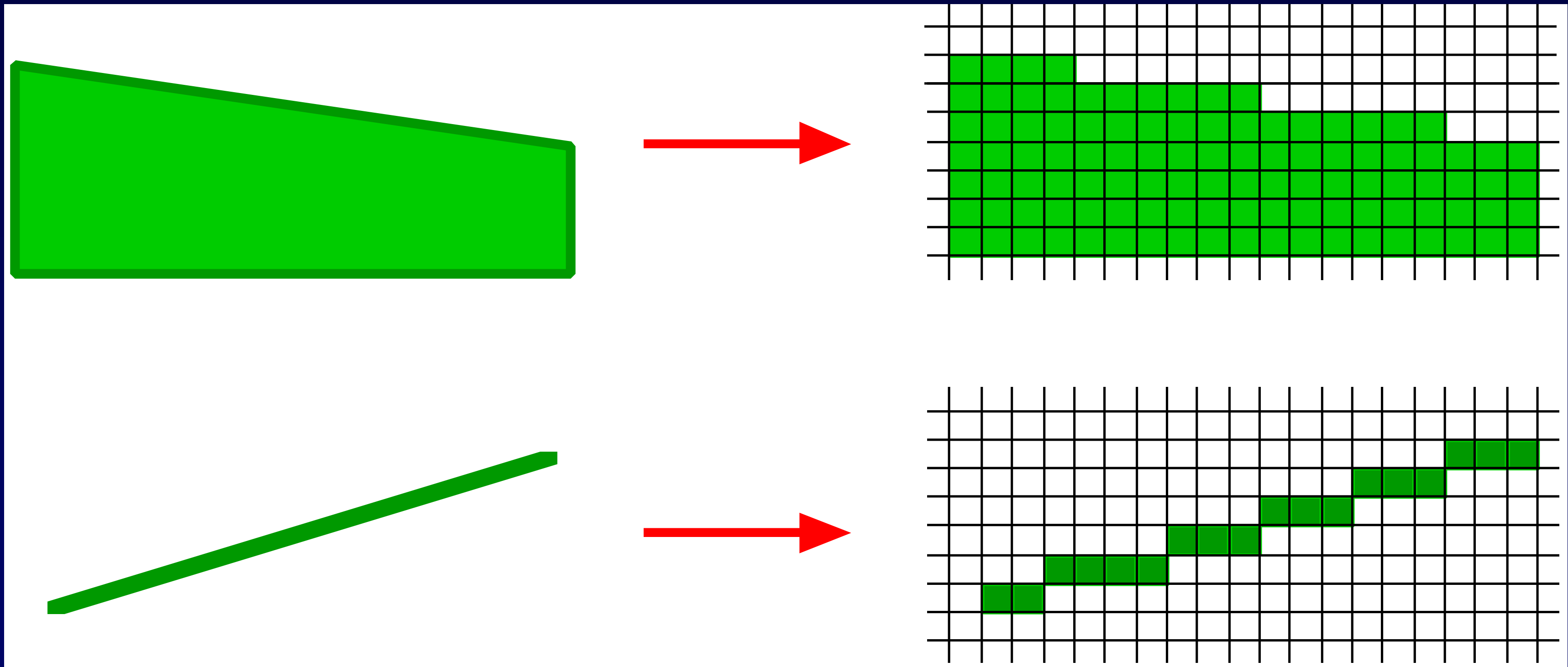
- Probleme mit Rasterisierung
- Probleme mit diskreten Daten



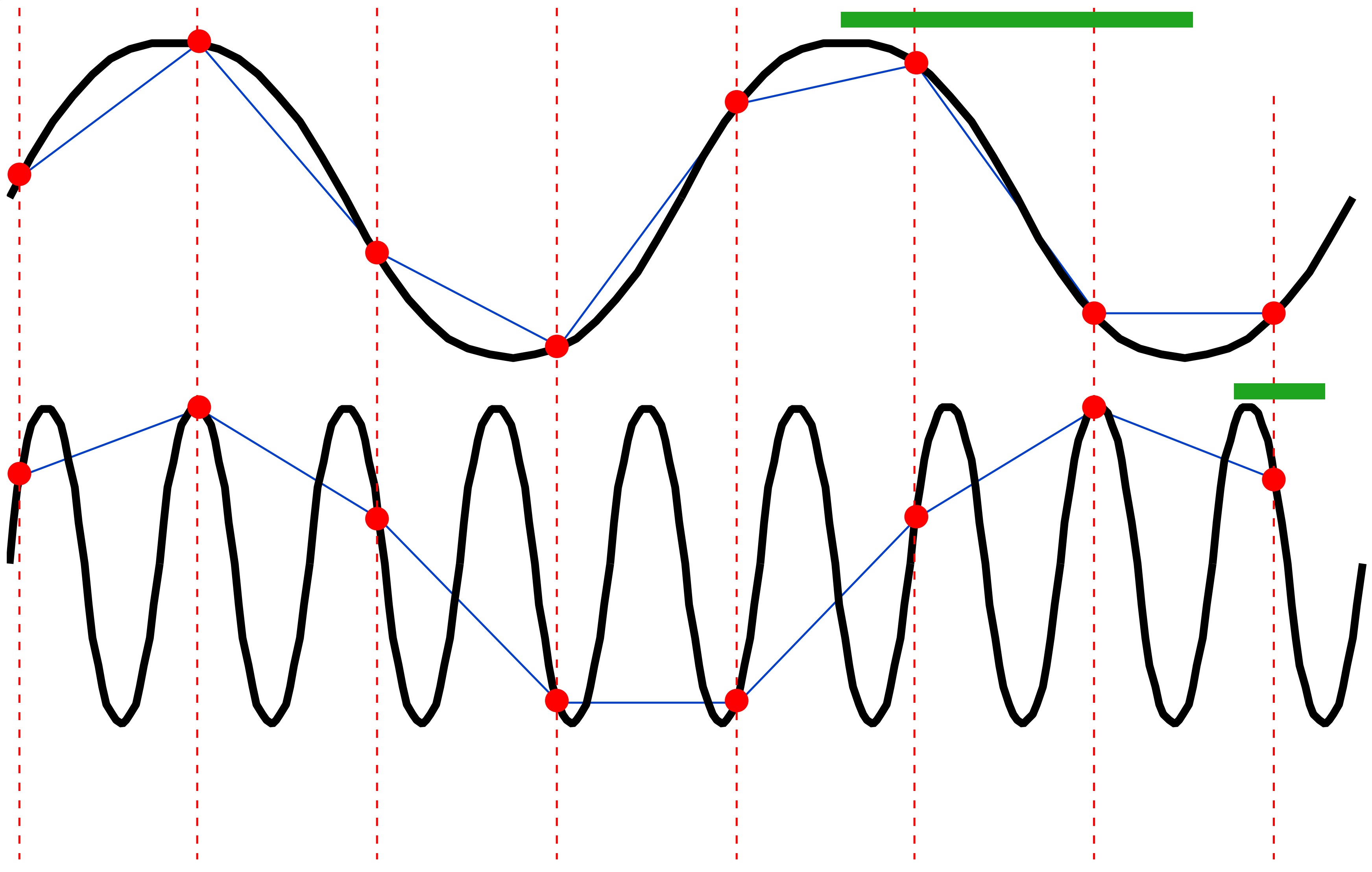
Aliasing – Beispiele

Aliasing: entsteht durch sampling

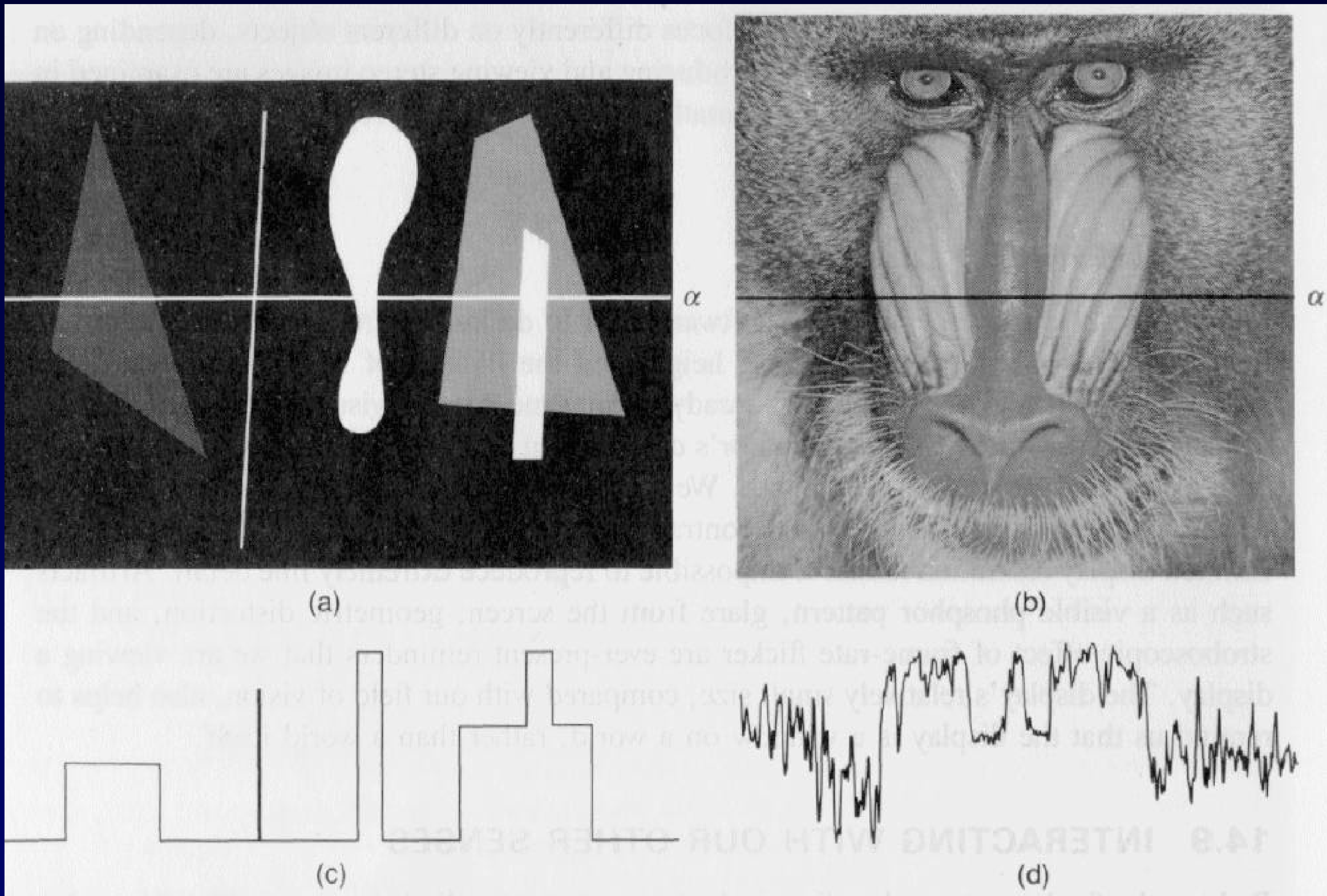
Sampling: abtasten analoger Daten



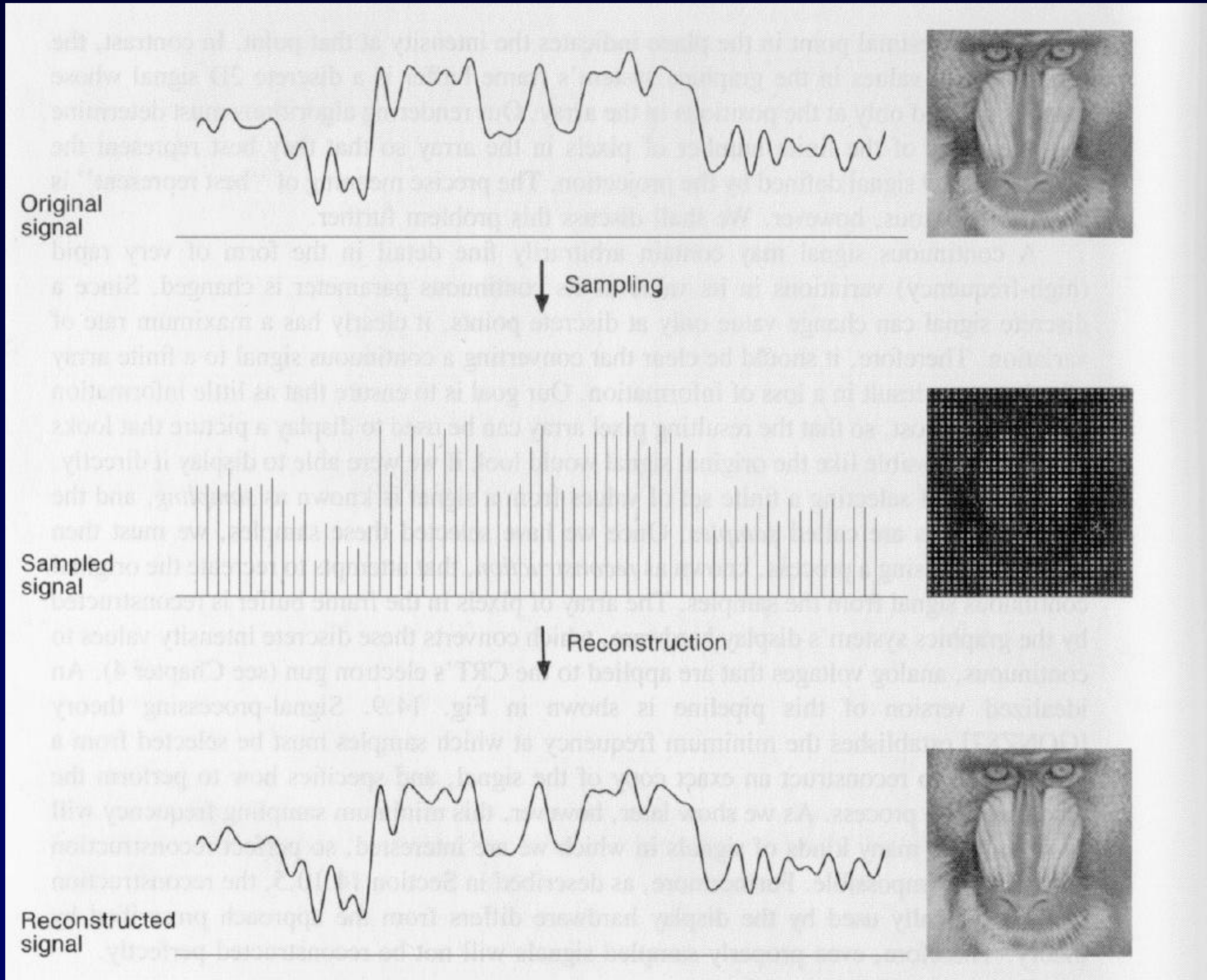
Sampling – Beispiel



Signale in der Computergraphik



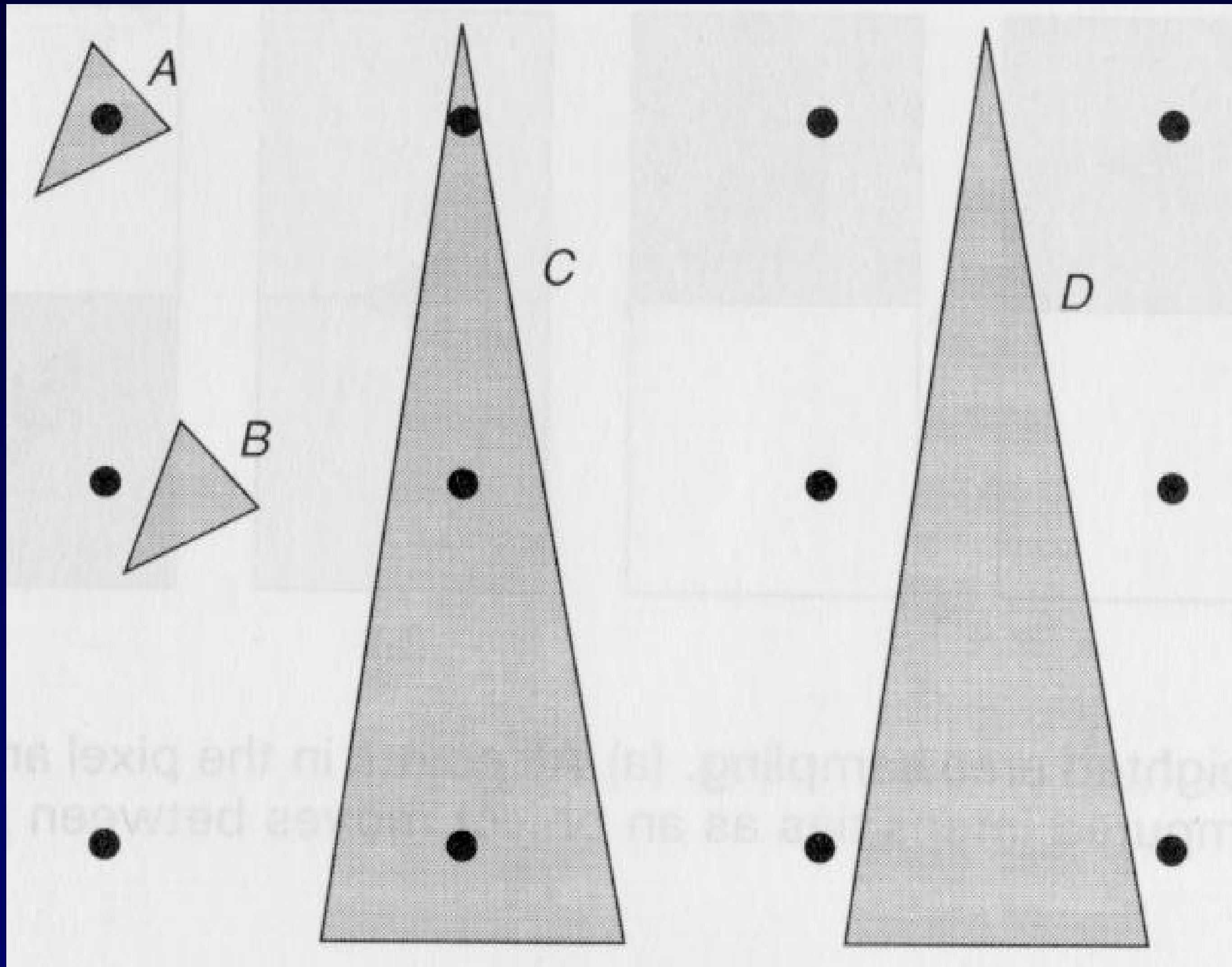
Sampling & Rekonstruktion



Sampling-Probleme: Aliasing

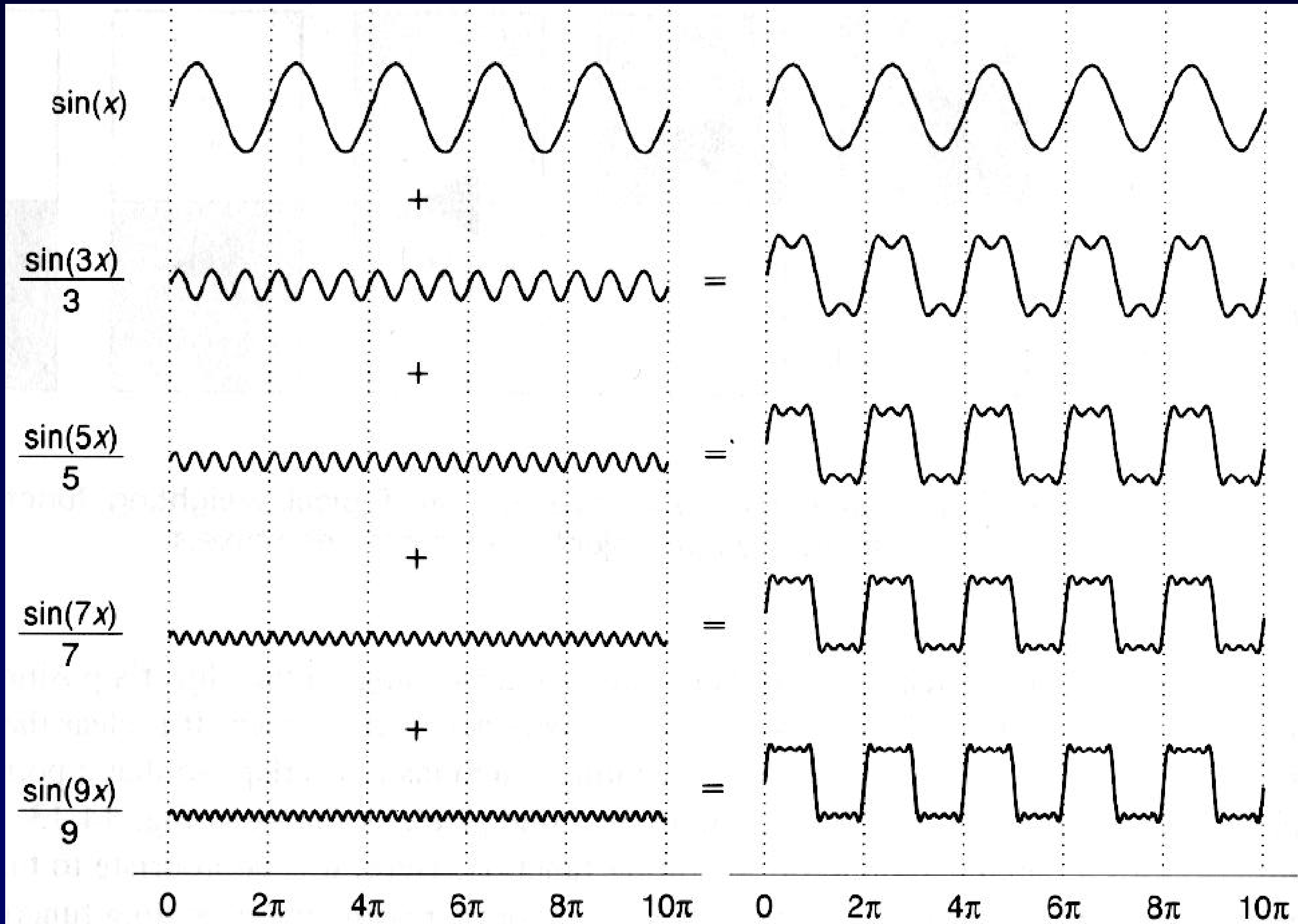
Je kleiner ein Detail,

desto eher nicht korrekt!

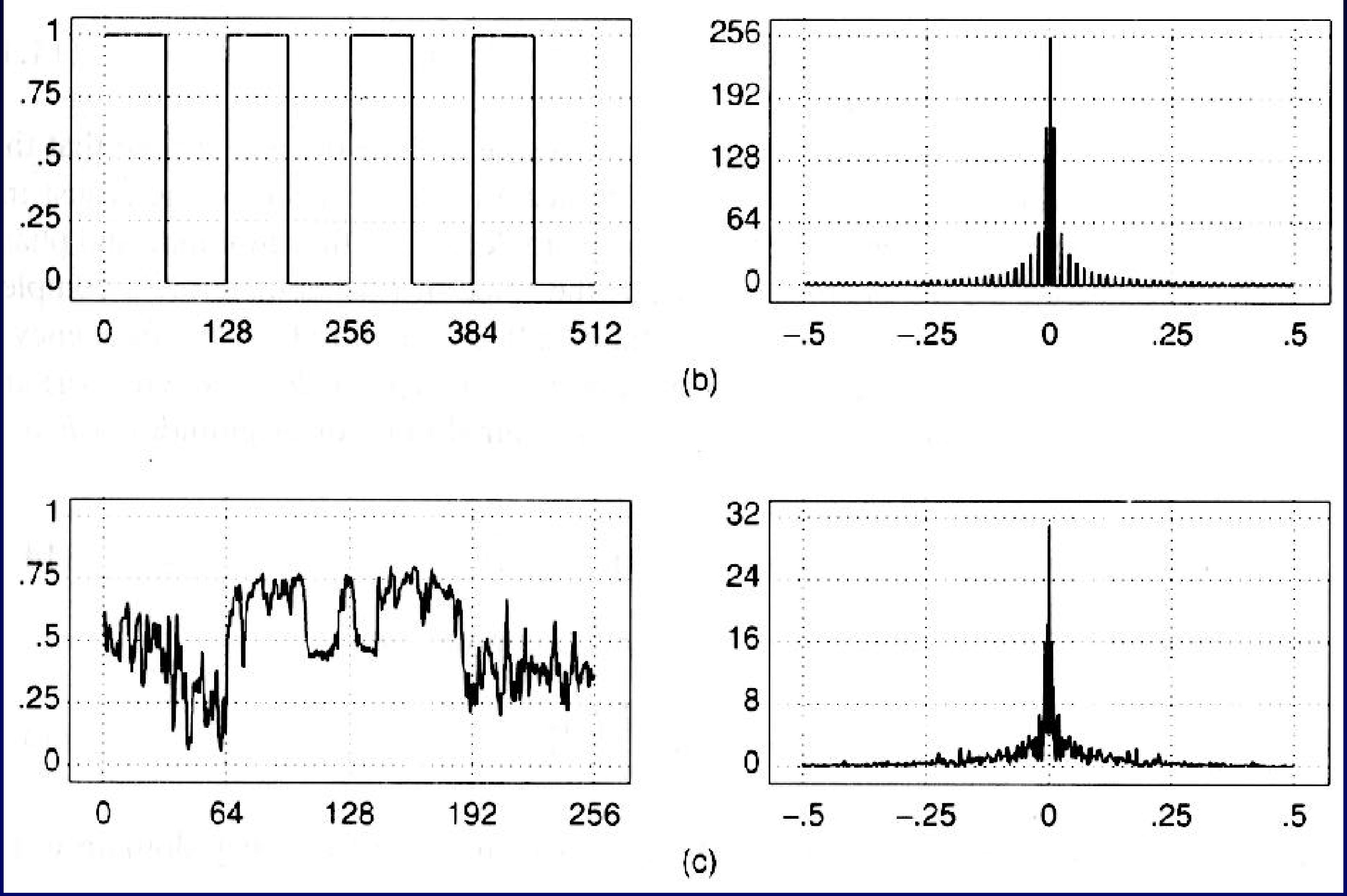


Signalmodellierung mit Wellen

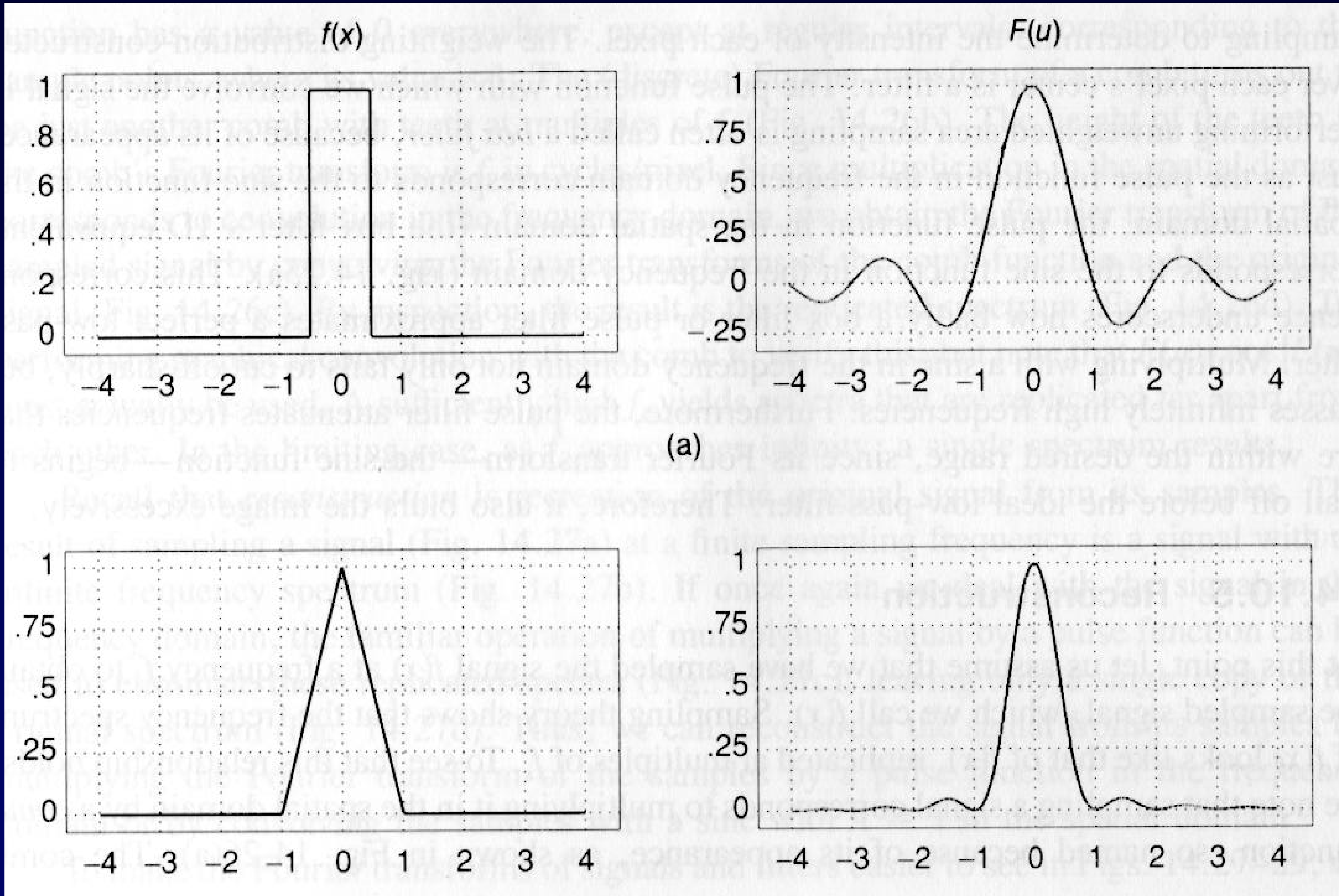
Bsp.: box



Spektren: Beispiele



Wichtige Paare (Ort/Frequenz)



Fourier-Transformation

Mittel, um

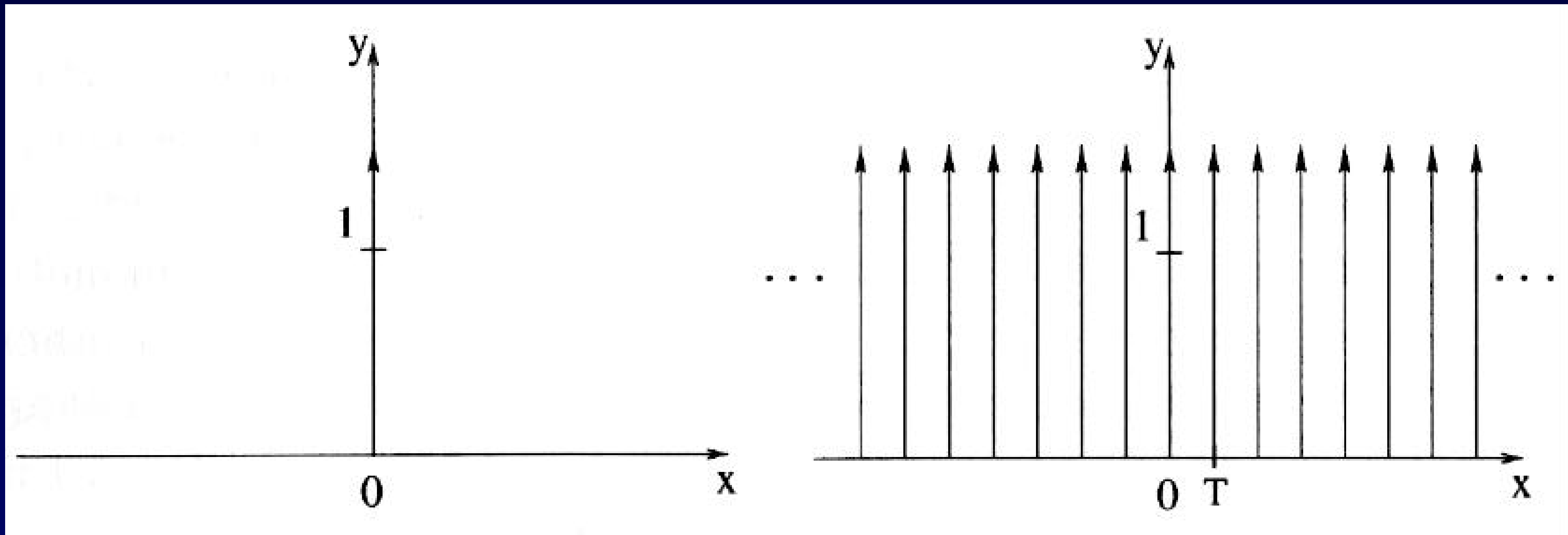
- das Spektrum eines Signals
- das Signal zu einem Spektrum

zu berechnen.

$$f(x) = \int_{\omega=-\infty}^{\infty} F(\omega) \cdot e^{2\pi j\omega x}\,\mathrm{d}\omega$$
$$F(\omega) = \int_{x=-\infty}^{\infty} f(x) \cdot e^{-2\pi j\omega x}\,\mathrm{d}x$$

$$f(x) = \sum_{\omega=0}^{N-1} F(\omega) \cdot e^{2\pi\pi j\omega/N}$$
$$F(\omega) = \frac{1}{N} \cdot \sum_{x=0}^{N-1} f(x) \cdot e^{-2\pi\pi j\omega/N}$$

Wichtige Funktionen



Dirac-Impuls **Kamm-Funktion**

Dazugehörige Spektren

Impuls-Funktion: Spektrum ≡ 1

- alle Frequenzen enthalten

Kamm-Funktion: Spektrum = Kamm!

- Je weiter der Kamm im Ortsraum,
- desto enger der Kamm im Frequ.-Raum!

$$\mathrm{comb}_{\mathrm{T}}(x) \Leftrightarrow \mathrm{comb}_{1/\mathrm{T}}(?\,)$$

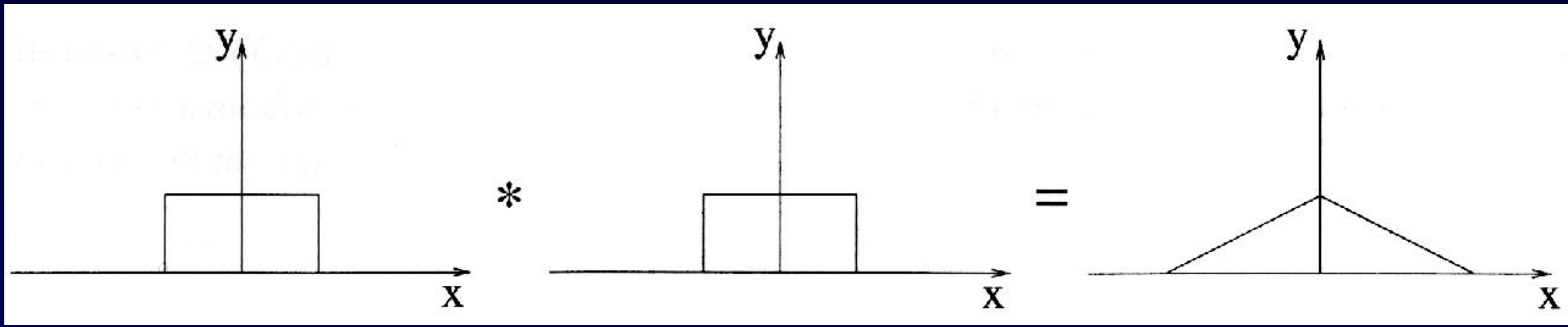
Wichtige Operation: Faltung

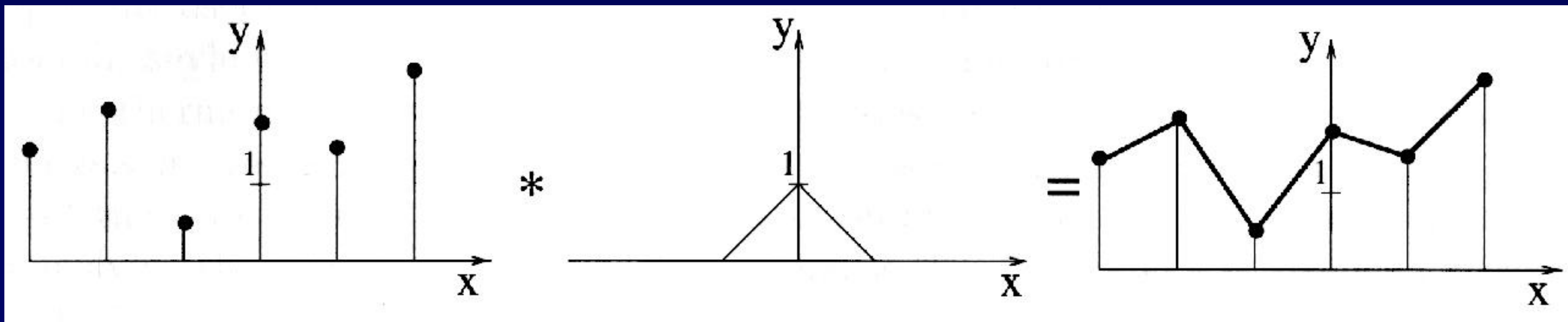
Faltung:

- h = f ο g
- Input-Funktion f, Filter-Funktion g
- h(x) = gewichtetes Mittel von f(t), t∈[x-w/2,x+w/2]

$$h(x) = (\, f \circ g\,)\big|_x = \int_{t=-\infty}^{\infty} f(t) \cdot g(x-t)\,\mathrm{d}t$$

Faltung – Beispiele





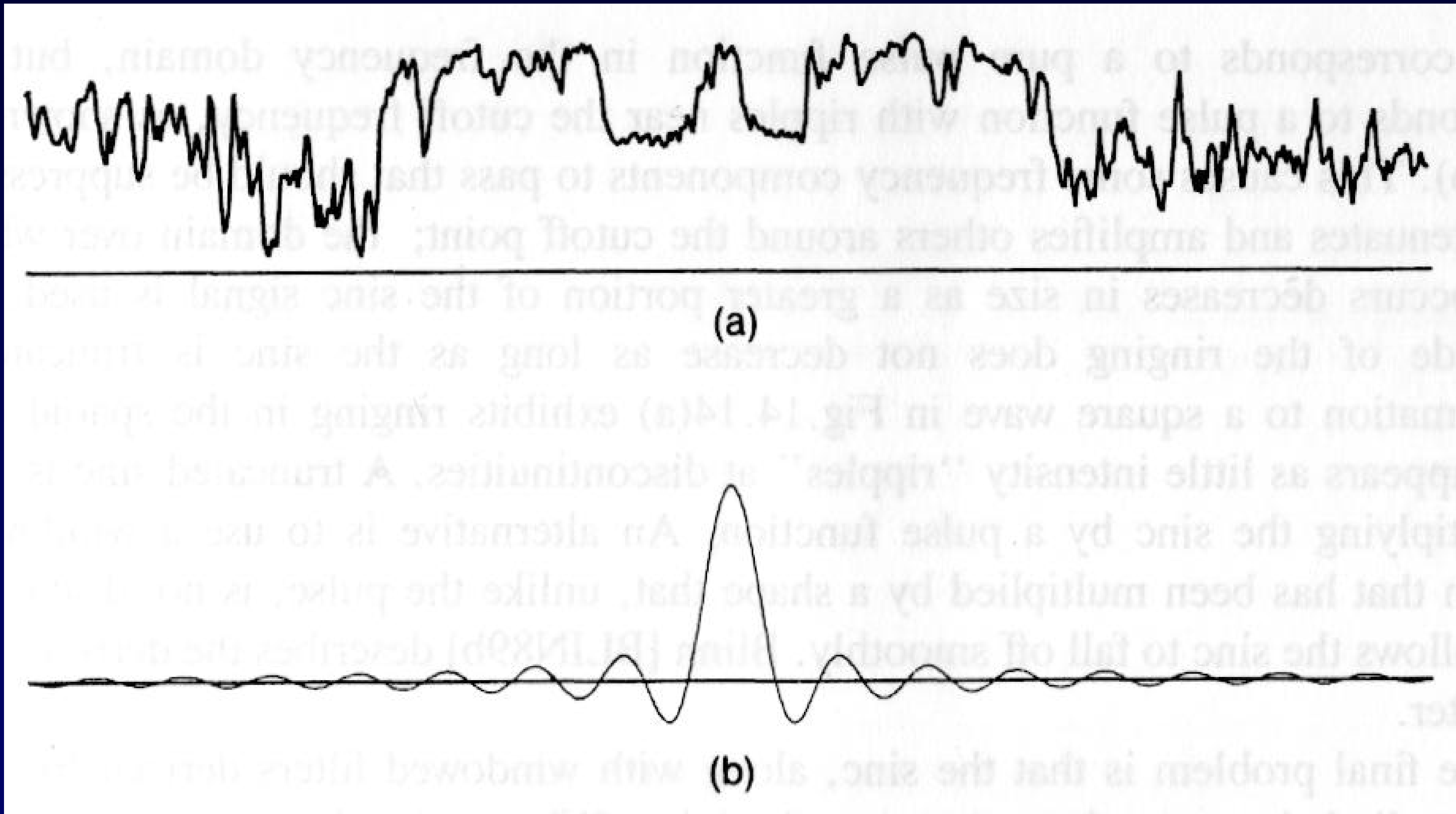
Faltung und Multiplikation

Ortsraum		Frequenzraum
Faltung	$\Leftrightarrow$	Multiplikation
Multiplikation	$\Leftrightarrow$	Faltung

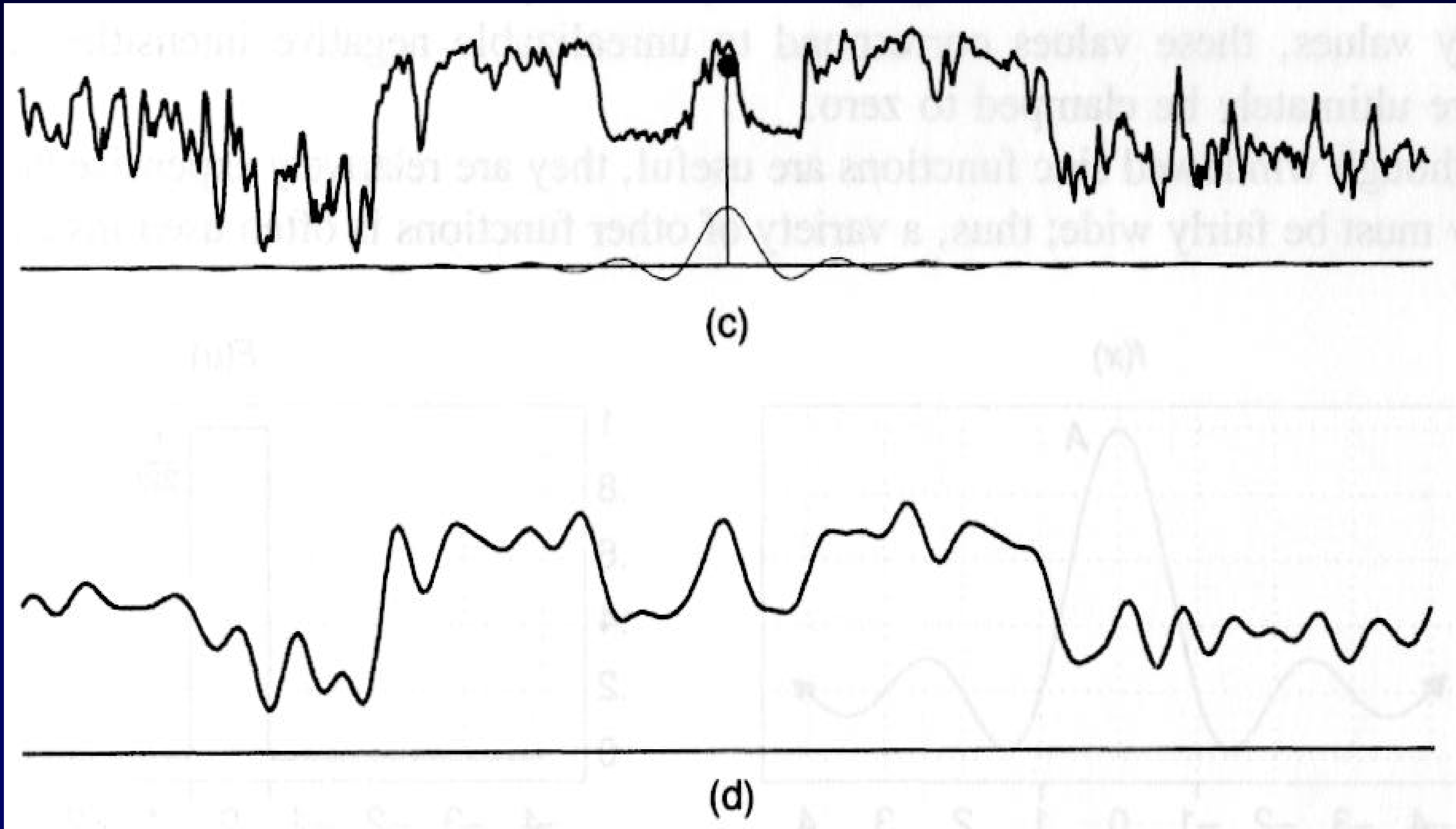
$$f \circ g = F \cdot G$$

$$f \cdot g = F \circ G$$

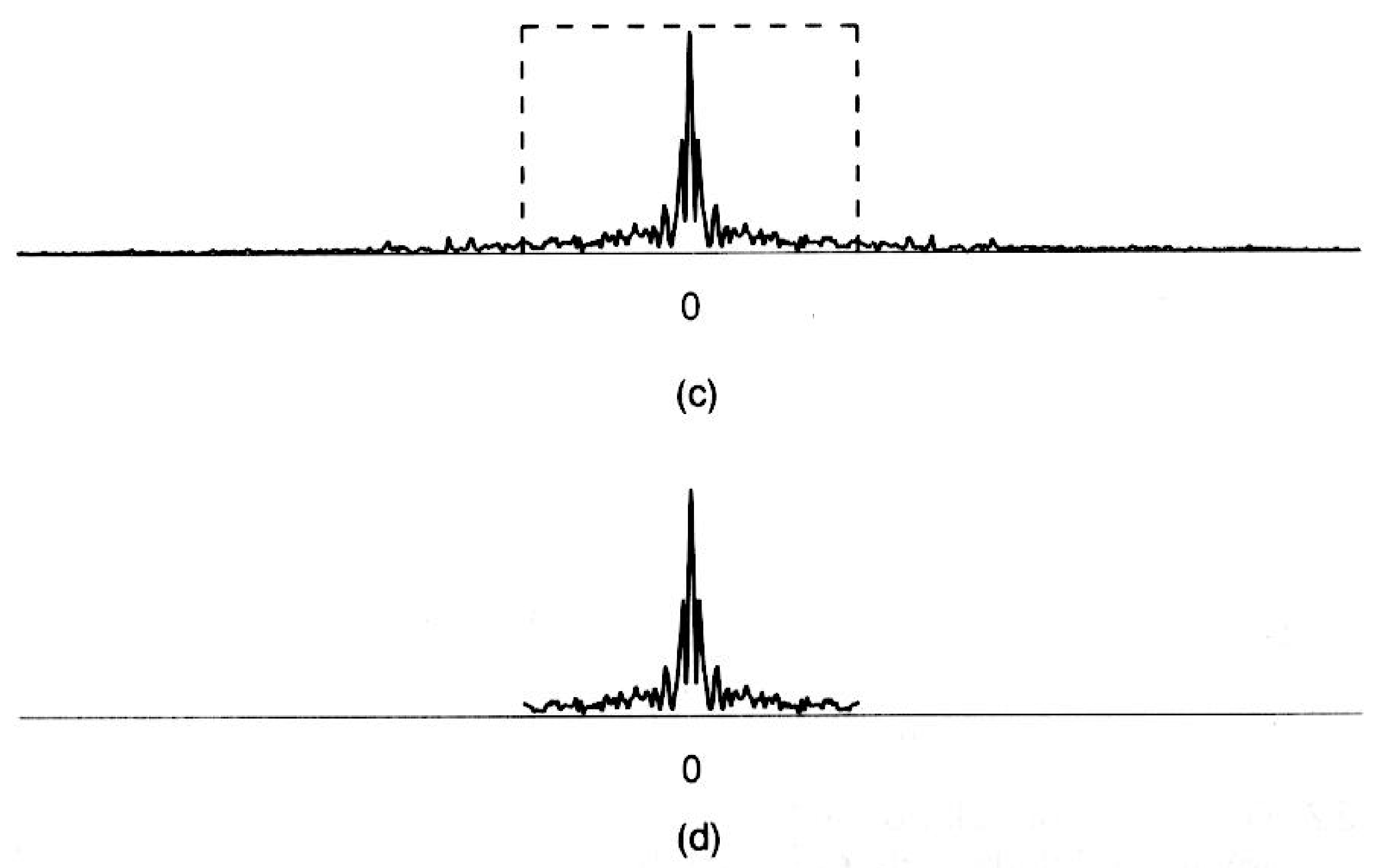
Low-Pass Prefiltering



Low-Pass: Faltung mit sinc



Low-Pass im Frequenzraum



Sampling

Sampling:

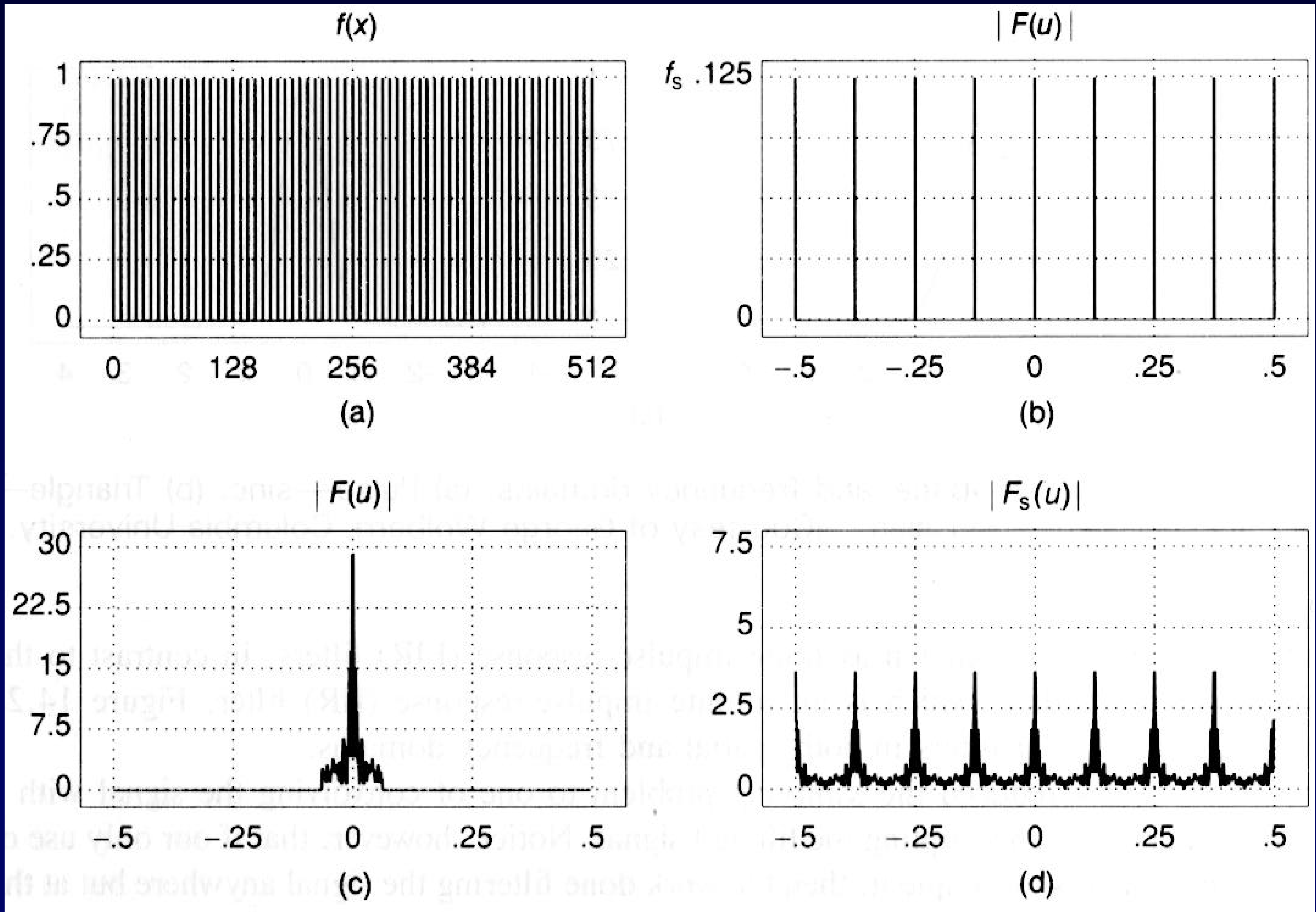
- Repräsentation durch Beispiele
- Uniform sampling: Beispiele (samples) regelmäßig organisiert (Gitter)
- Multiplikation mit Kamm-Funktion (Ort)

$$h(x) \quad = \quad f(x) \cdot \mathrm{comb}_T(x)$$

- Faltung mit Kamm (Frequenzraum)

$$H(\omega) \quad = \quad F(\omega) \circ \mathrm{Comb}_{\frac{1}{T}}(\omega)$$

Spektrum diskreter Funktionen



Sampling Theorem

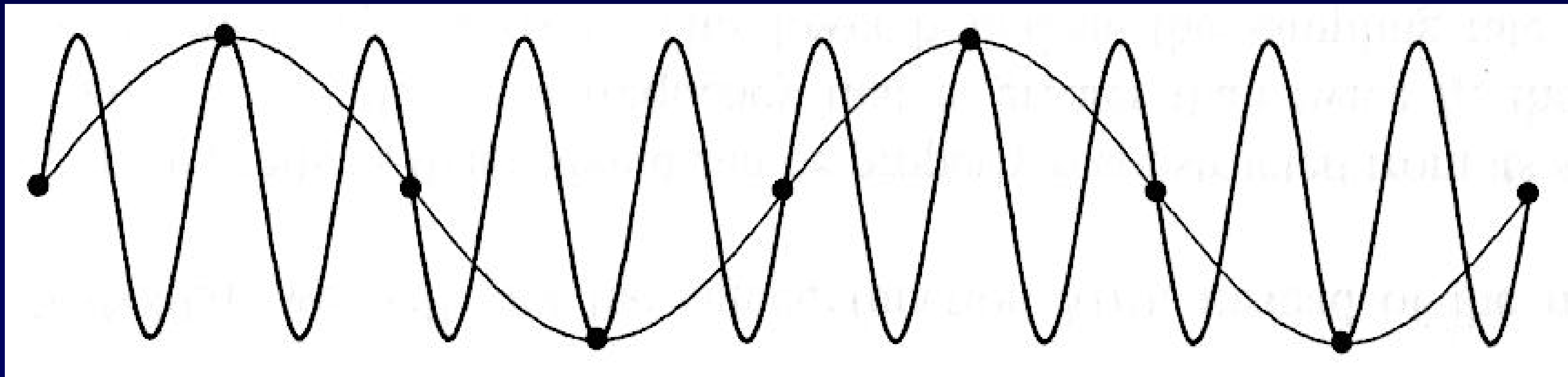
A function f which is

- band-limited and
- sampled above the Nyquist frequency

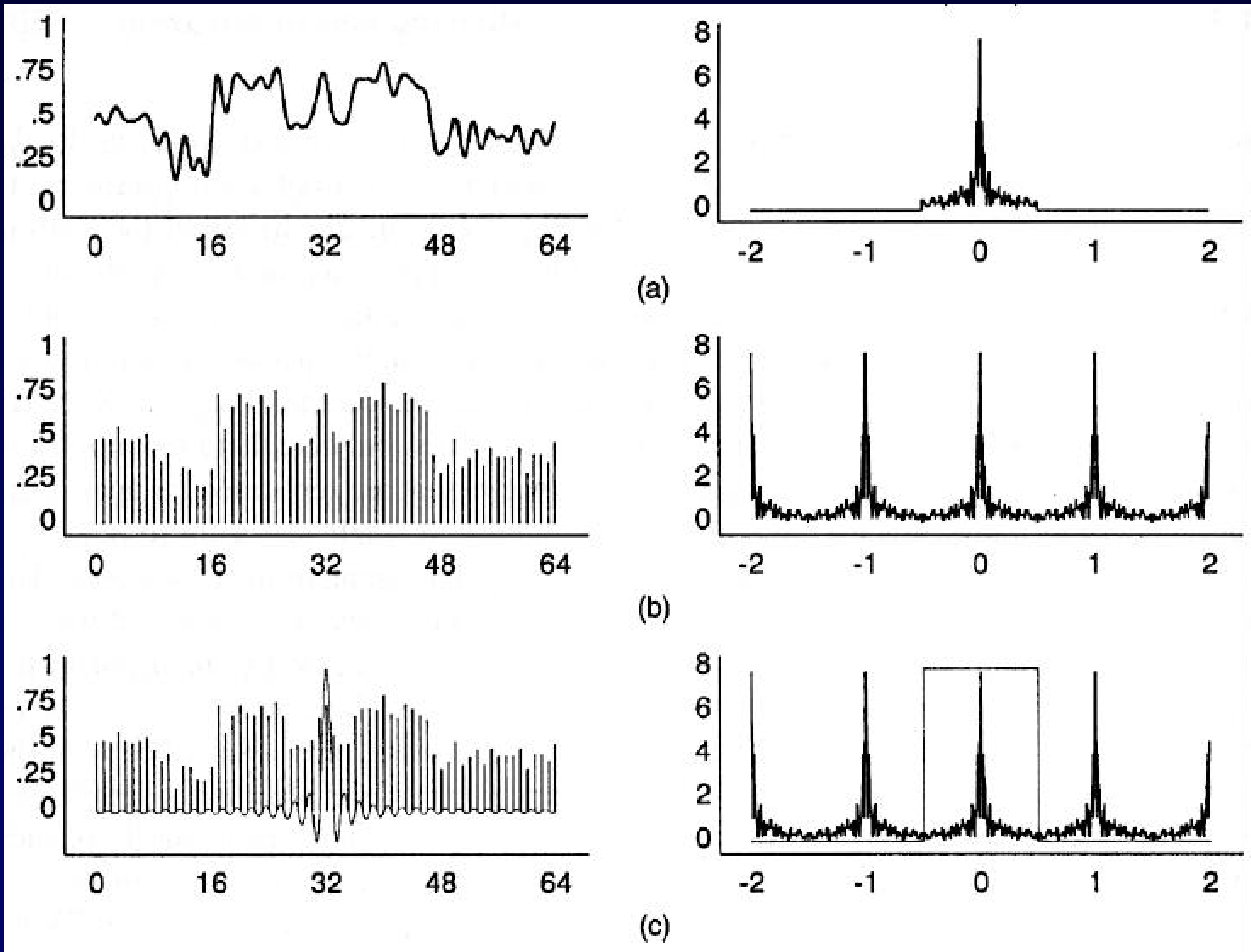
is completely determined by its samples.

Sampling / Nyquist

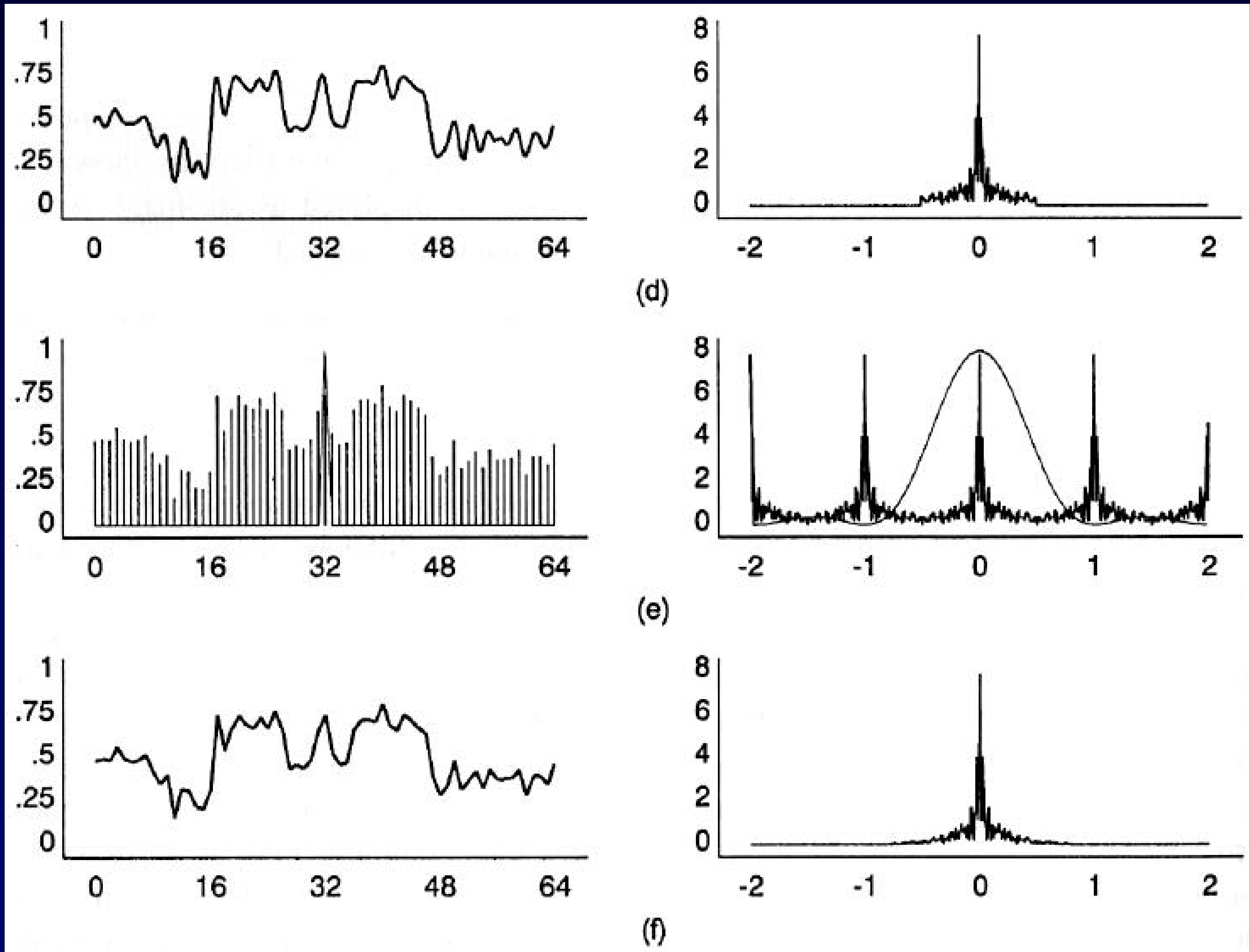
Sampling-Rate zu niedrig → aliasing



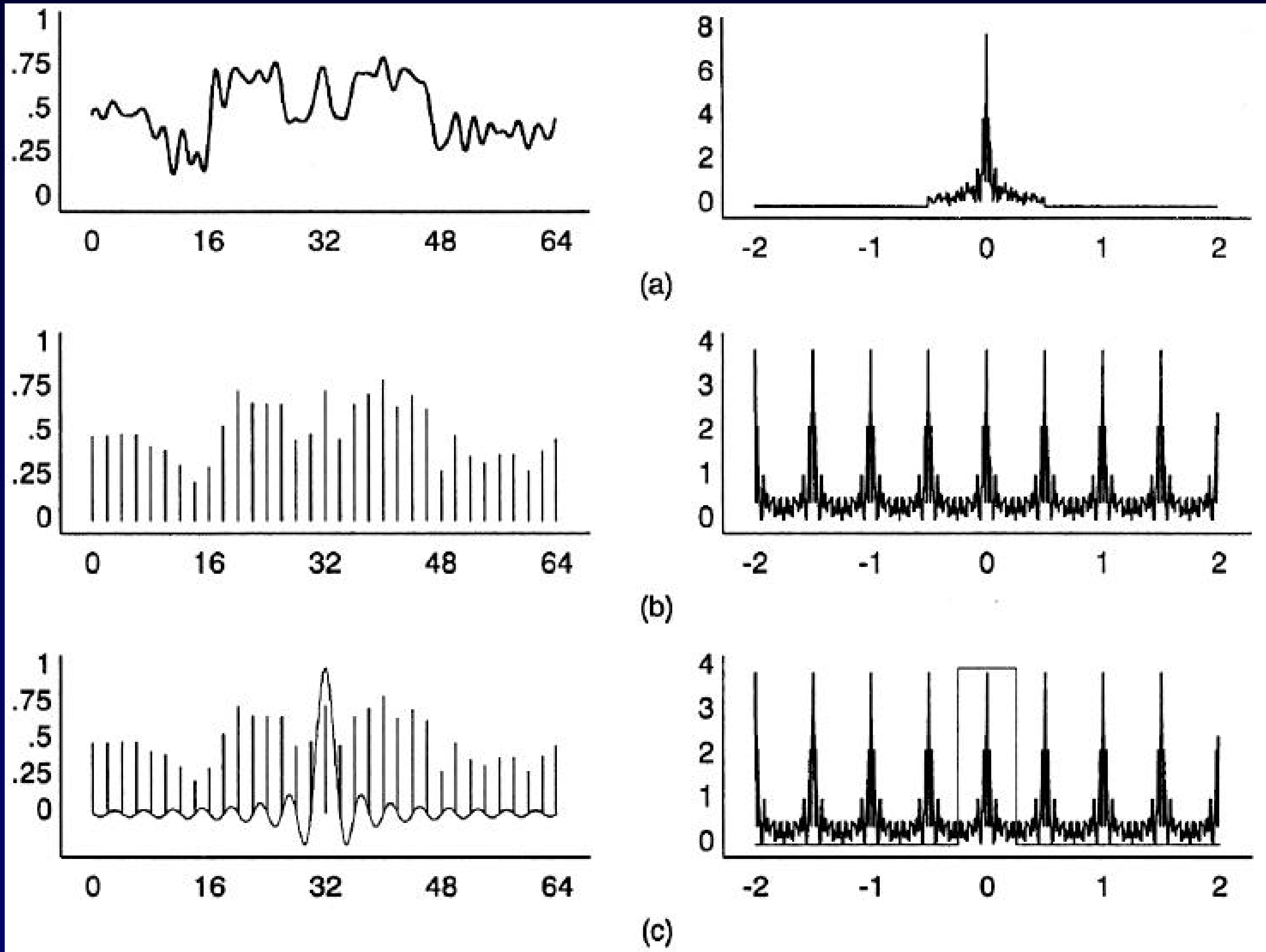
Passende Sampling-Rate



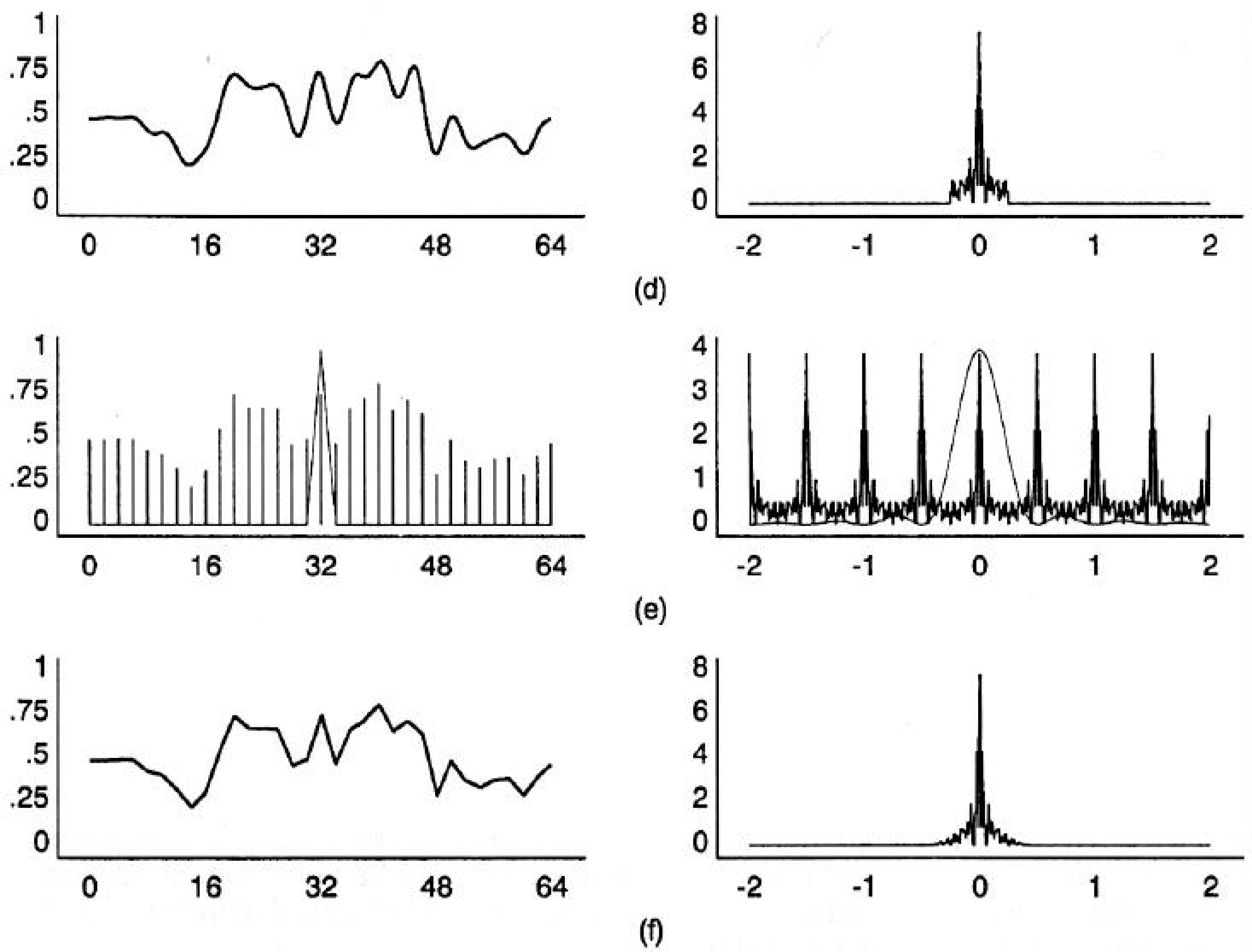
Passende Sampling-Rate (2)



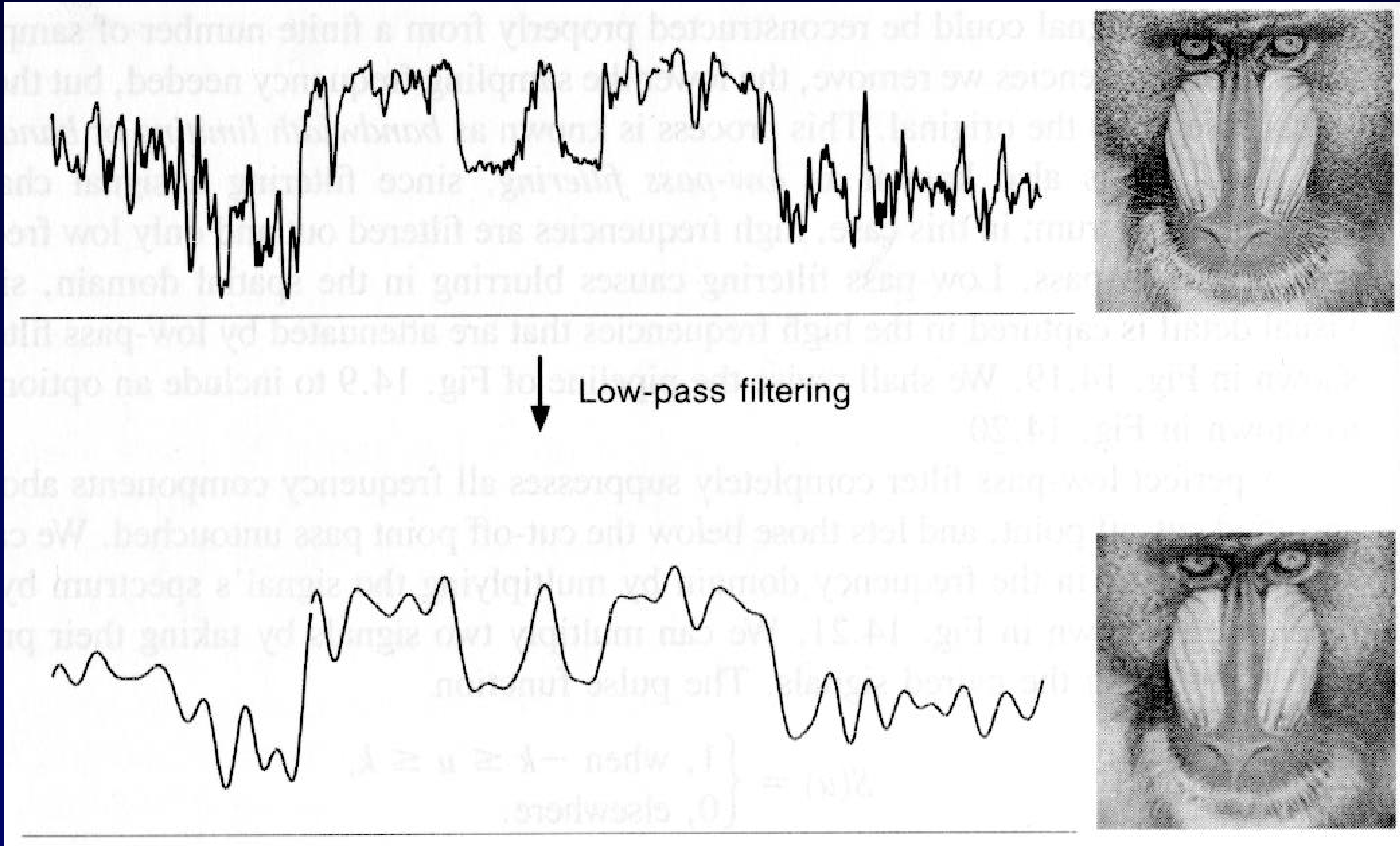
Unpassende Sampling-Rate



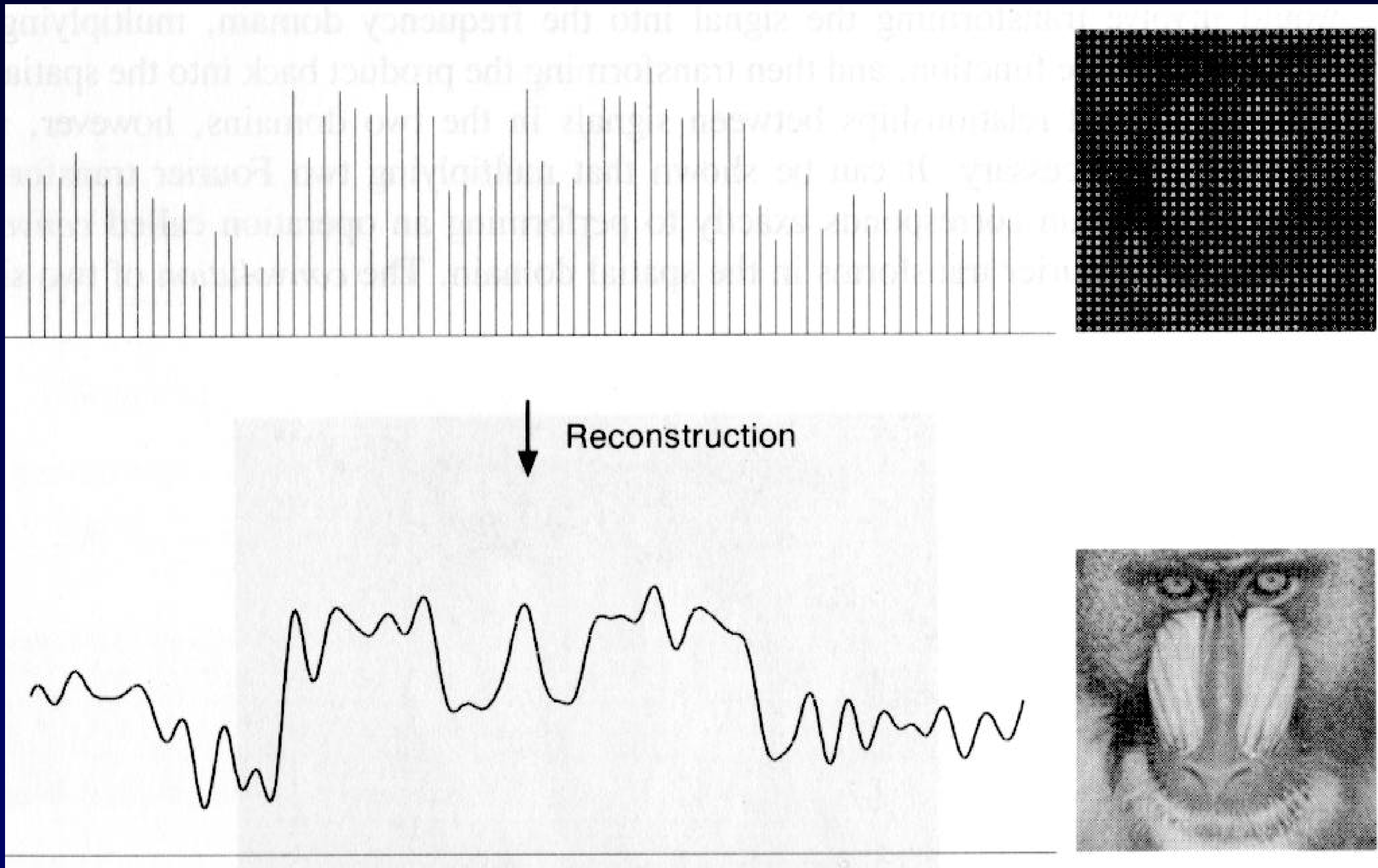
Unpassende Sampling-Rate



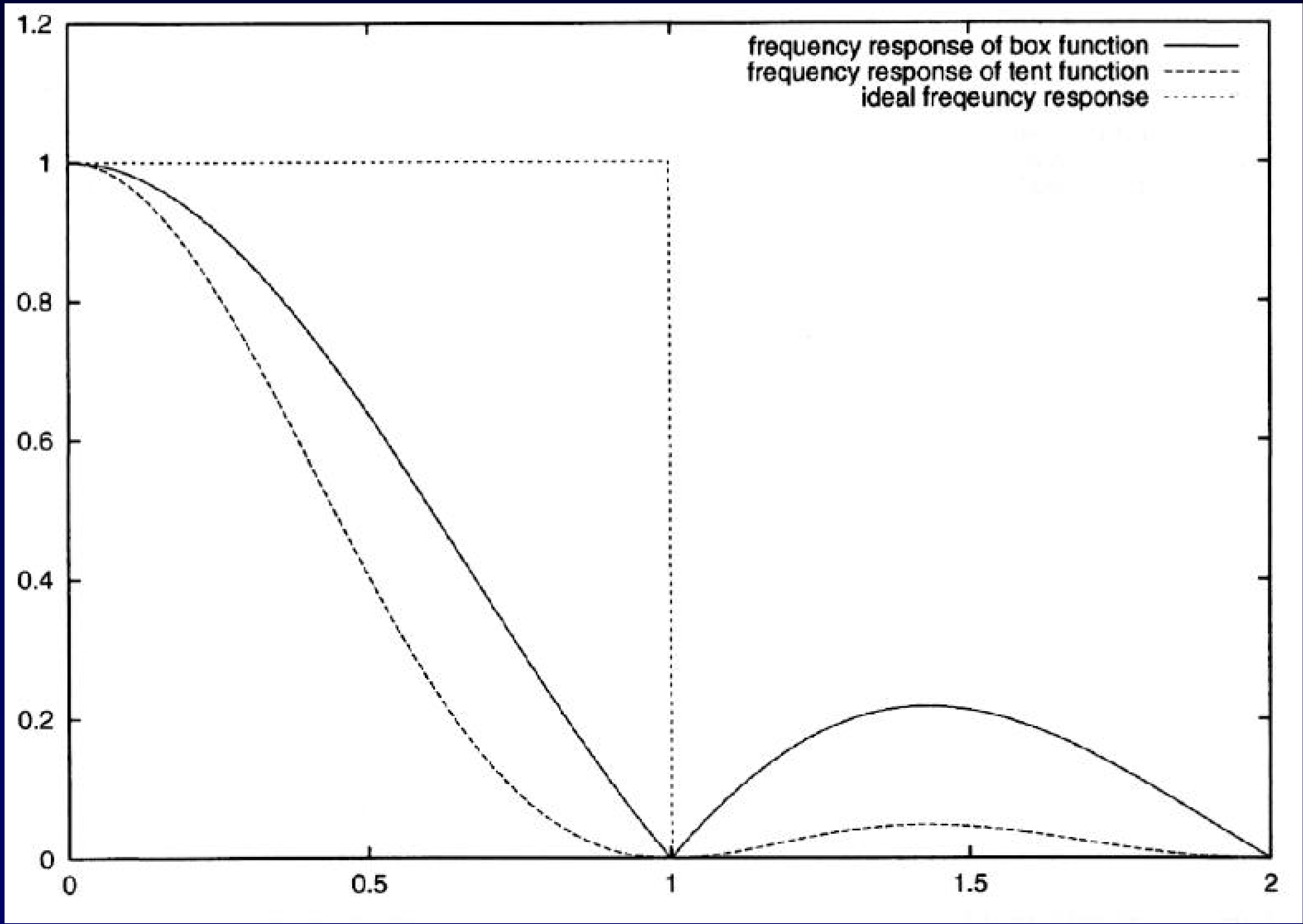
Prefiltering



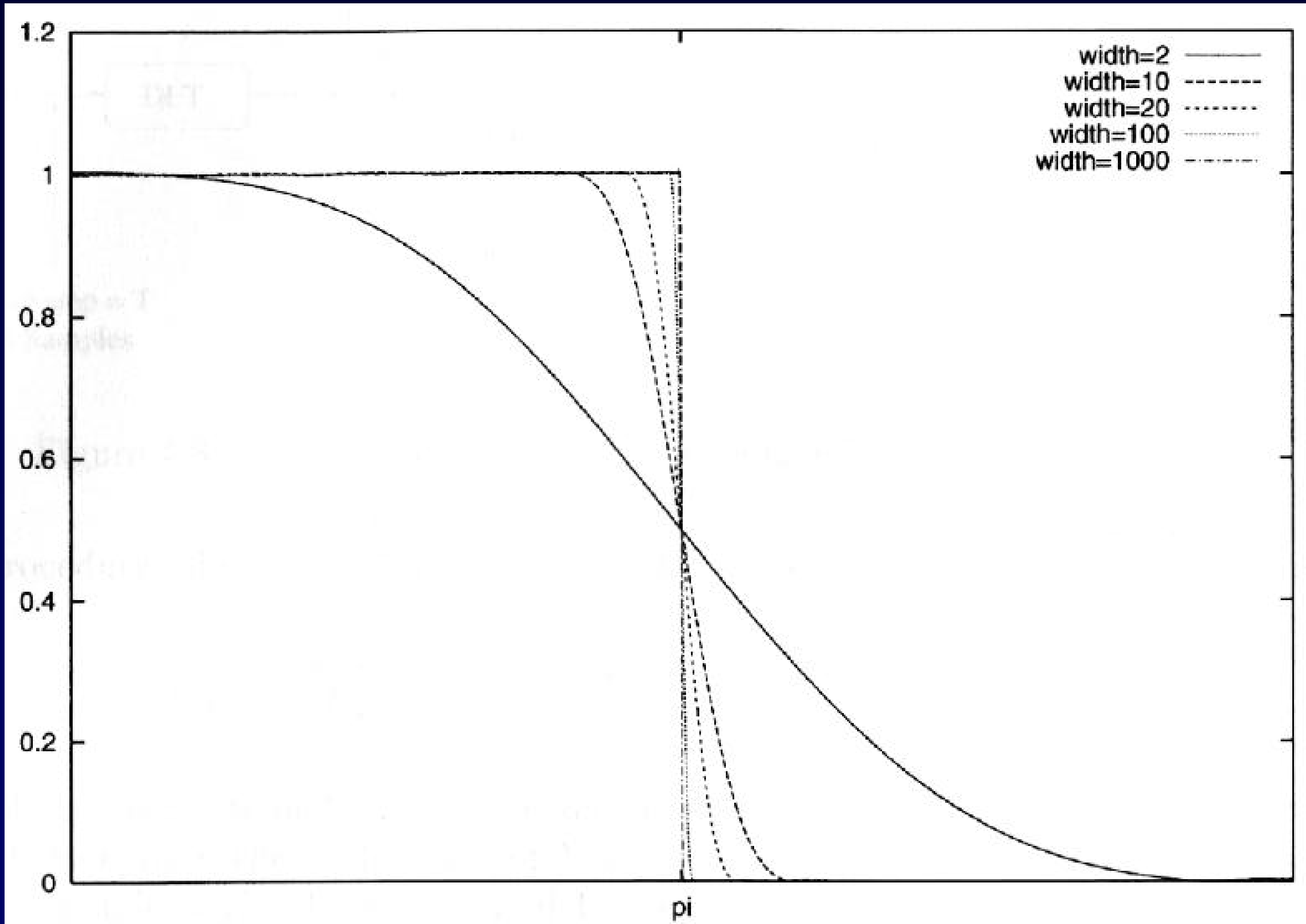
Rekonstruktion



Spektren von box, tent



Gefensterter sinc – Spektren



Aliasing – zwei Arten

Aliasing:

- hohe Frequ. mischen sich mit niedrigen
- doppelter Verlust:
  - hohe Frequ. weg
  - niedrige Frequ. falsch

Rekonstruktionsfehler

- Filter $\neq$ sinc
- Smoothing, ringing

Anti-Aliasing

Prefilterting

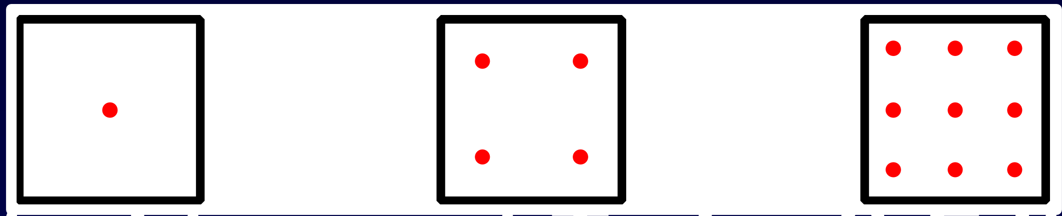
- Signal **vor sampling** durch low-pass!
- Ergebnis (band-limited) kann korrekt abgetastet werden (Nyquist-Frequenz)

Supersampling

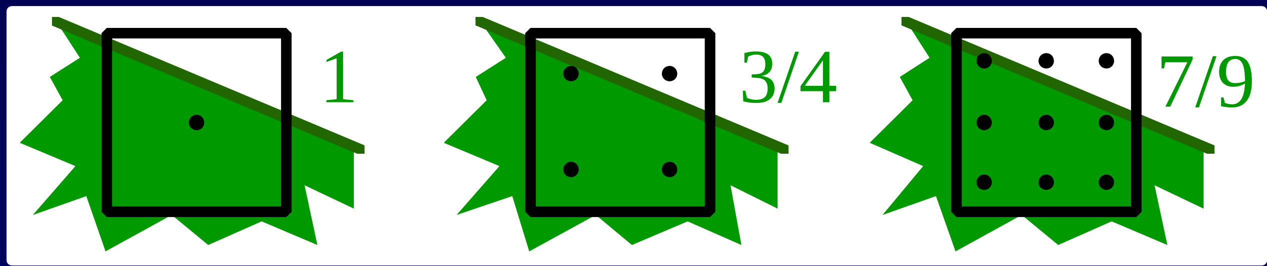
- Erhöhen der sampling-Rate

Supersampling

Sampling mit erhöhter Frequenz

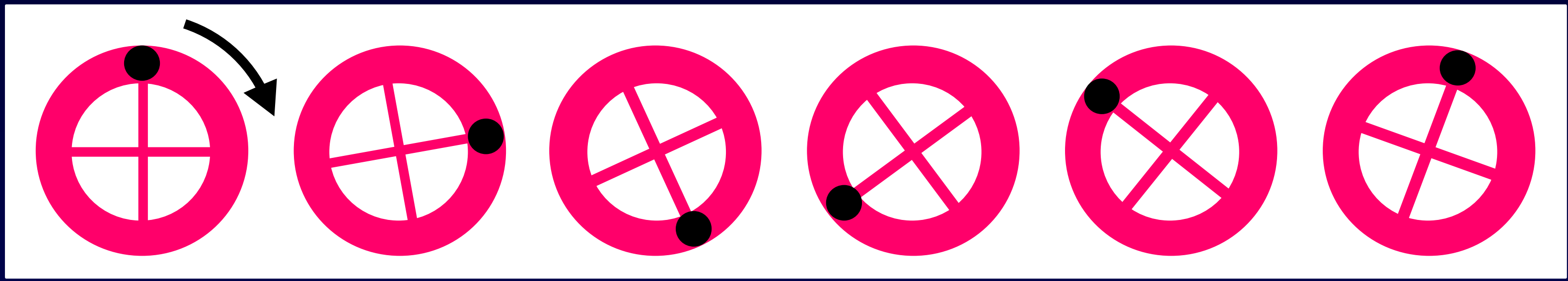


Rekonstruktion mit Tiefpaß-Filter



Aliasing im Zeitbereich

Umkehrung der Drehrichtung



Worming