

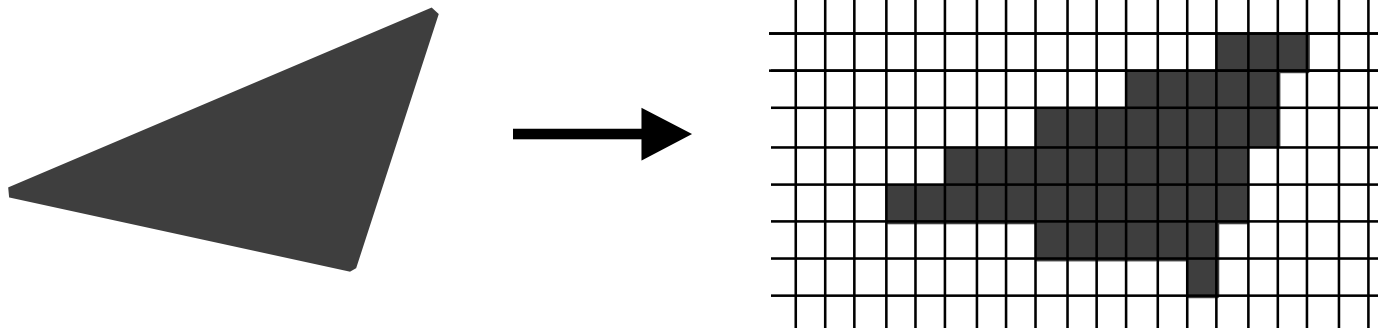
Teil 3: über Aliasing

Sampling, Rekonstruktion

Motivation

Aliasing – was ist das?

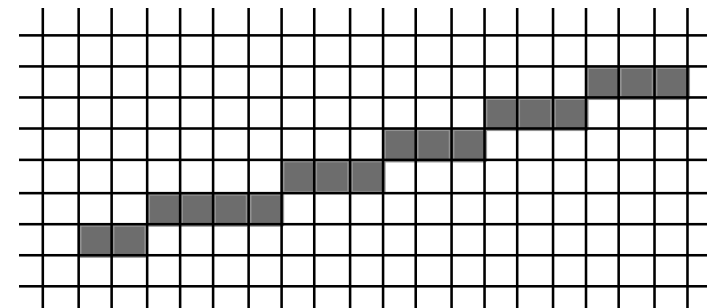
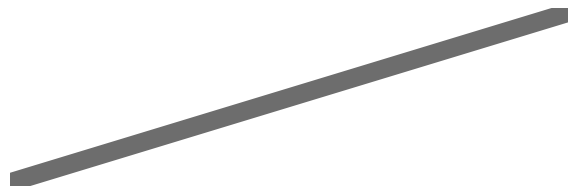
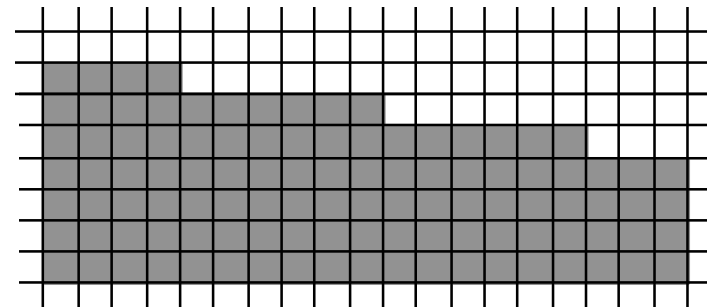
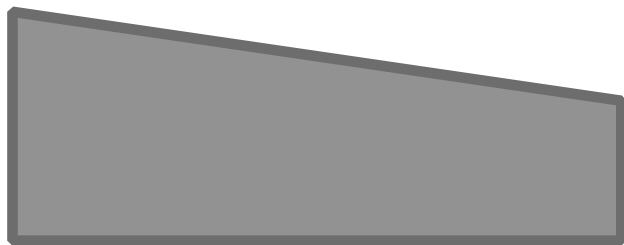
- ◆ Probleme aus Ungleichung
kontinuierlich \neq diskret
- ◆ Probleme mit Rasterisierung
- ◆ Probleme mit diskreten Daten



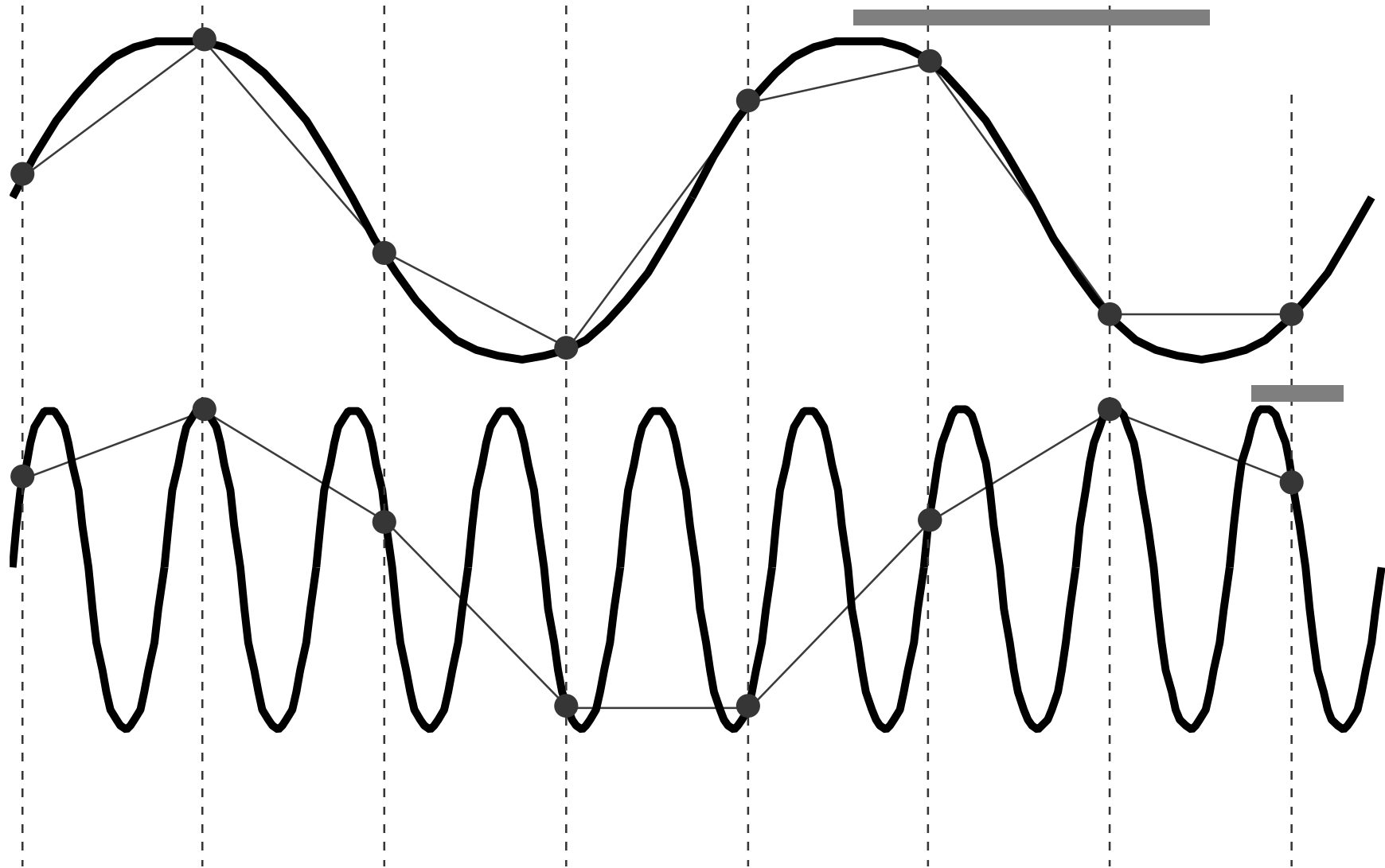
Aliasing – Beispiele

Aliasing: entsteht durch sampling

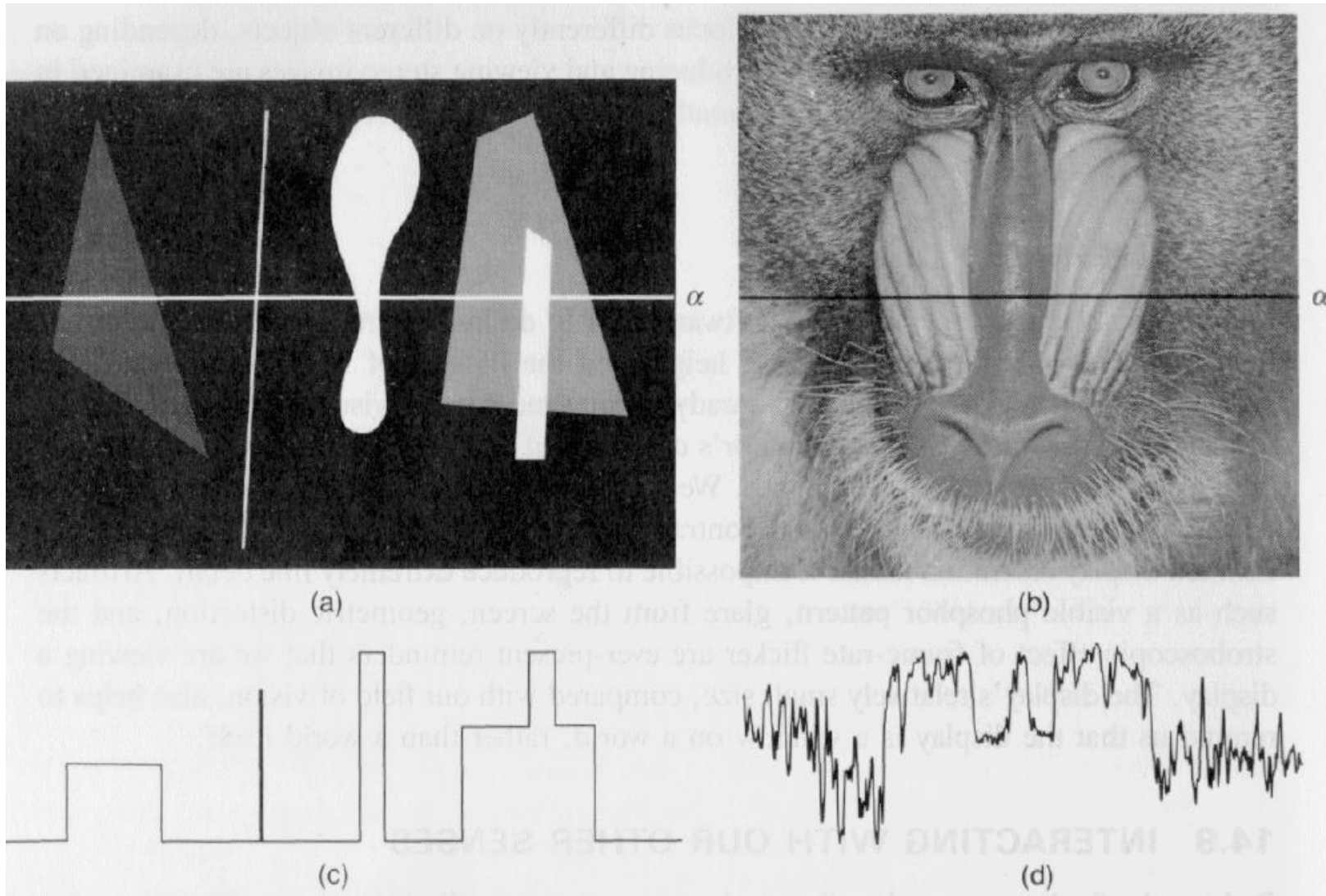
Sampling: abtasten analoger Daten



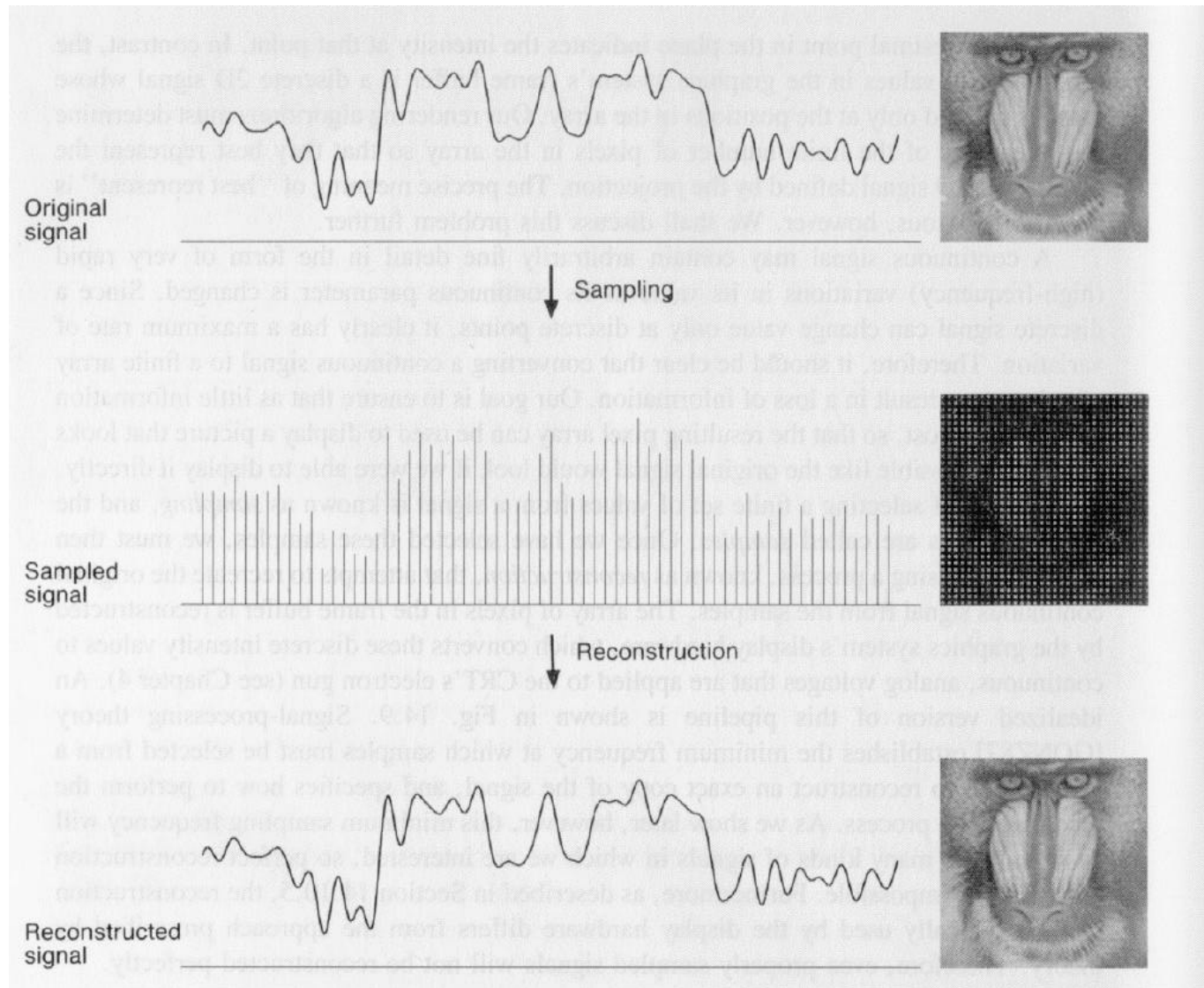
Sampling – Beispiel



Signale in der Computergraphik

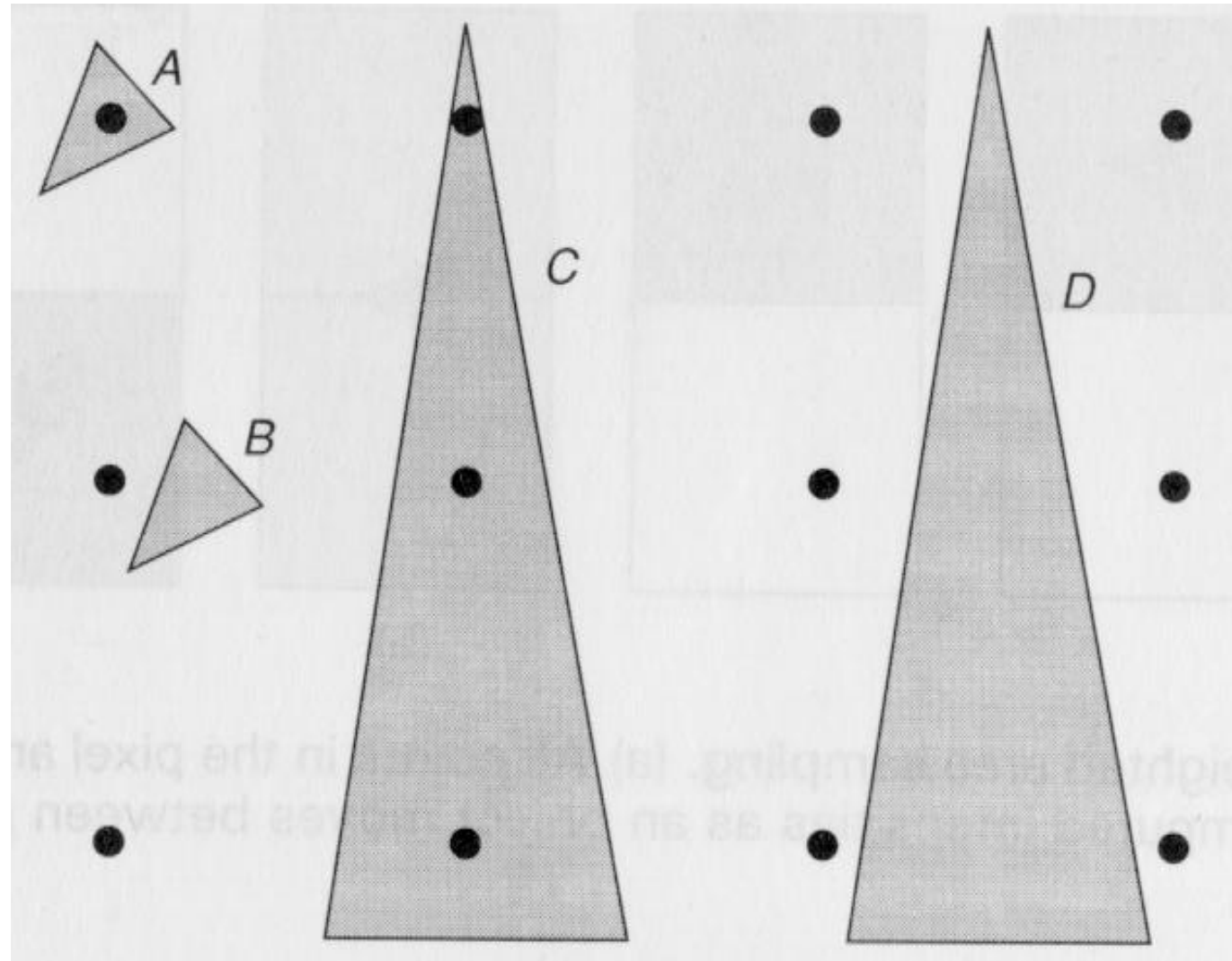


Sampling & Rekonstruktion



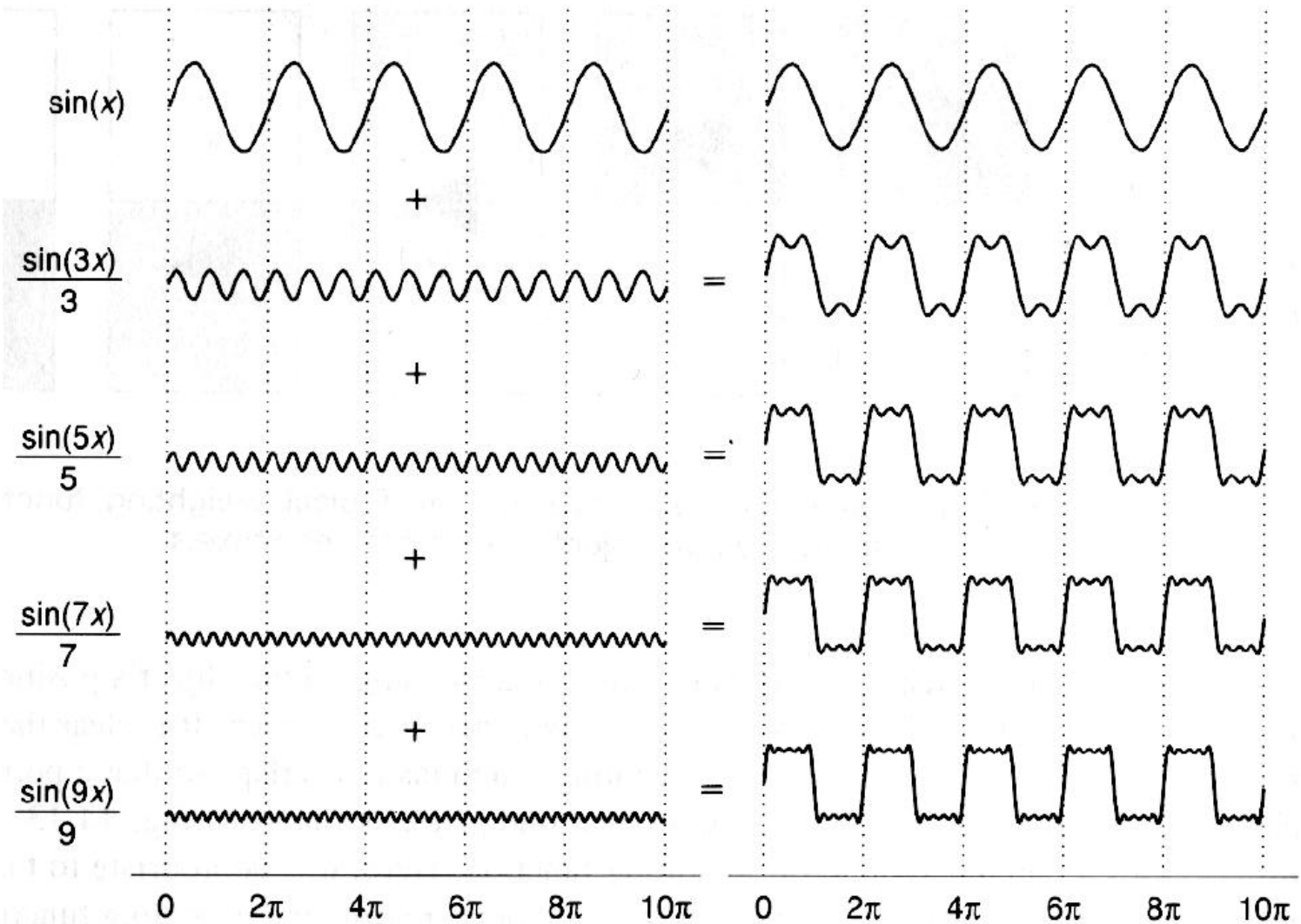
Sampling-Probleme: Aliasing

**Je
kleiner
ein
Detail,
desto
eher
nicht
korrekt!**

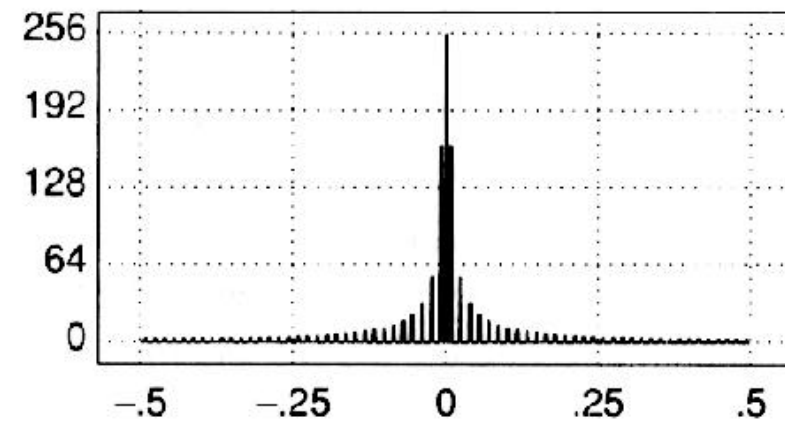
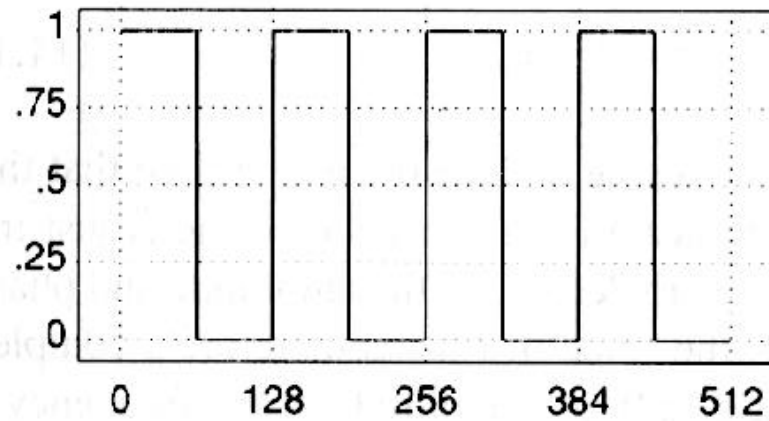


Signalmodellierung mit Wellen

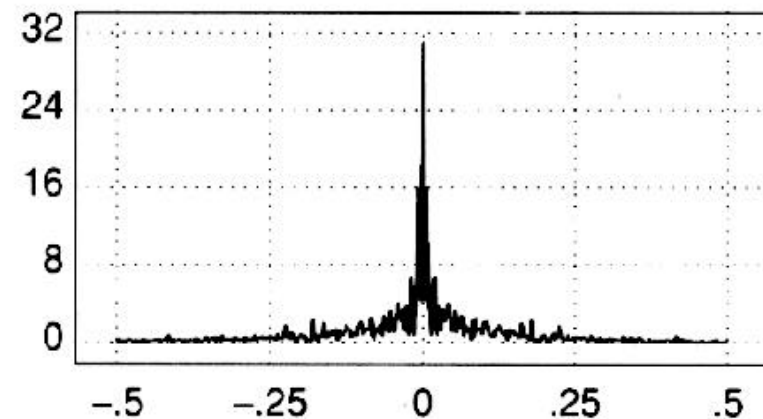
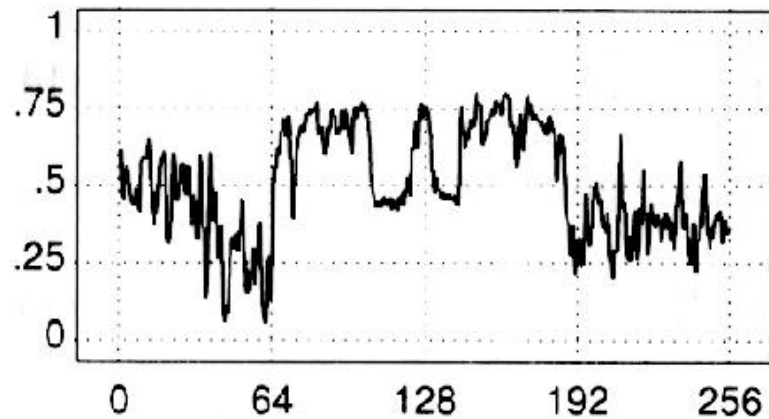
Bsp.:
box



Spektren: Beispiele

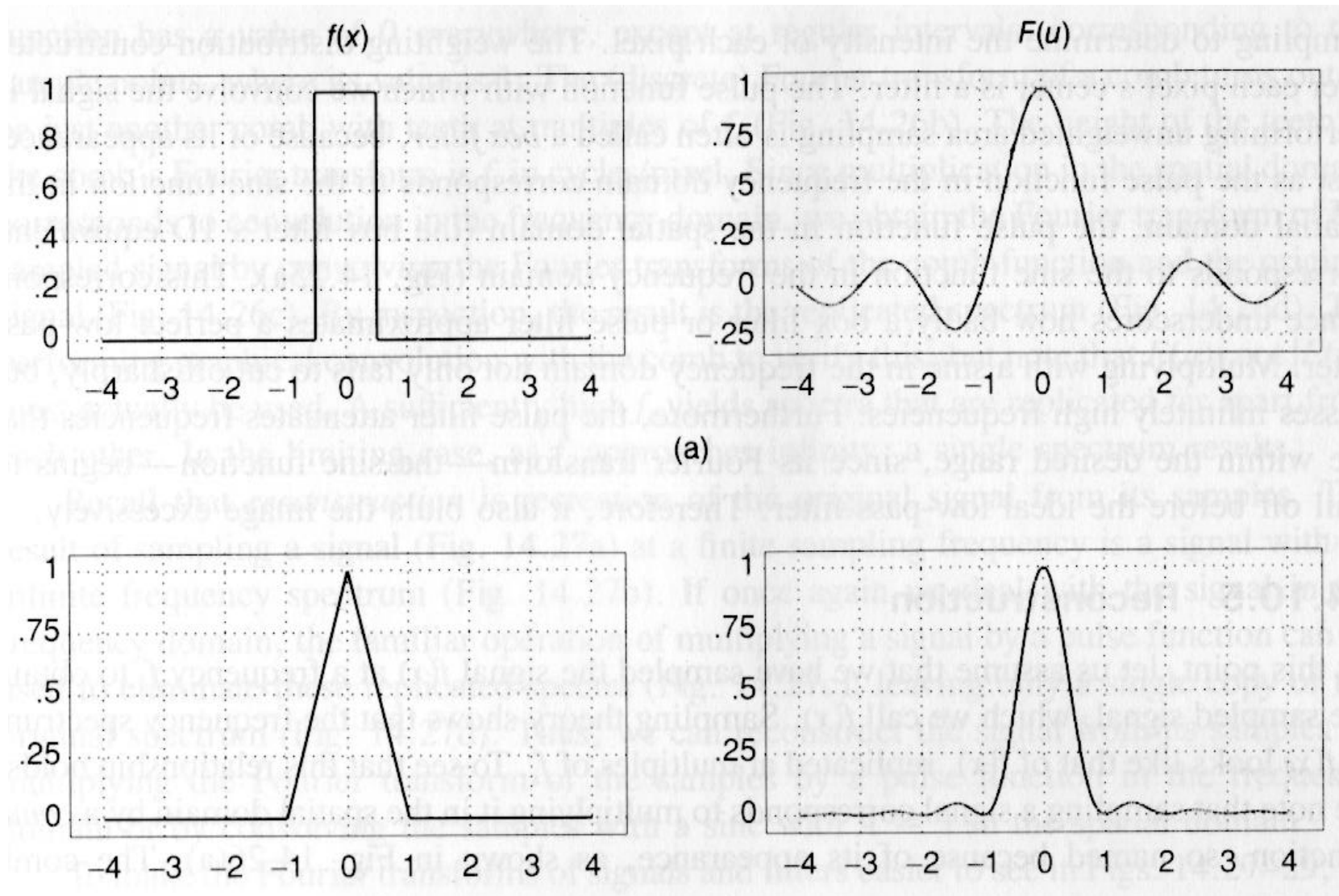


(b)



(c)

Wichtige Paare (Ort/Frequenz)



Fourier-Transformation

Mittel, um

- ◆ das Spektrum eines Signals
- ◆ das Signal zu einem Spektrum

zu berechnen.

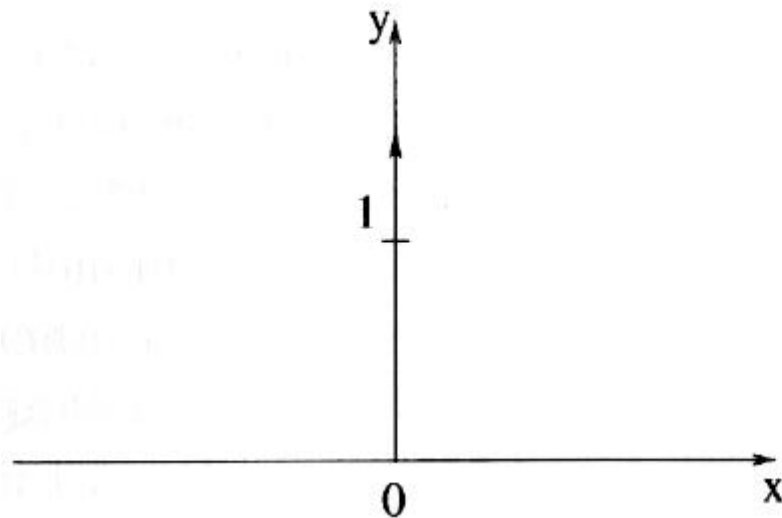
$$f(x) = \int_{\omega=-\infty}^{\infty} F(\omega) \cdot e^{2\pi j\omega x} d\omega$$

$$F(\omega) = \int_{x=-\infty}^{\infty} f(x) \cdot e^{-2\pi j\omega x} dx$$

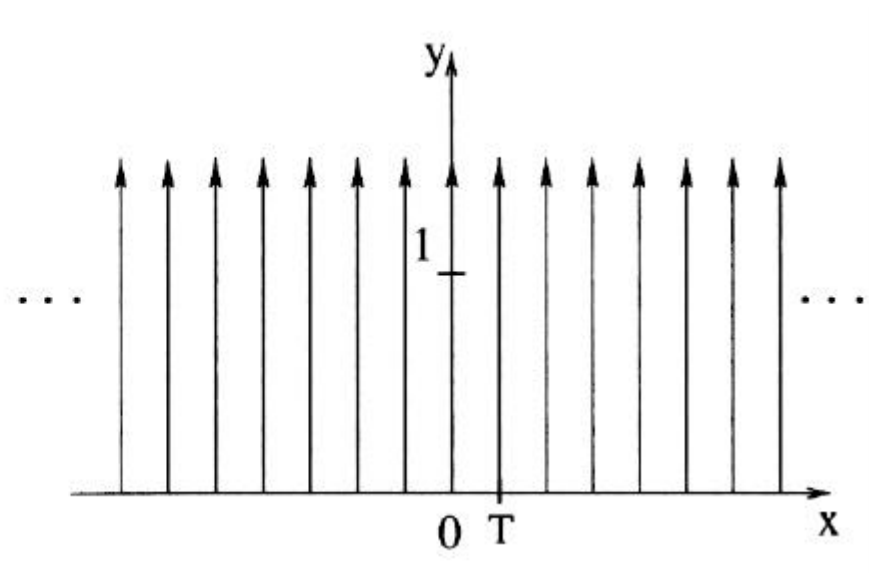
$$f(x) = \sum_{\omega=0}^{N-1} F(\omega) \cdot e^{2\pi j\omega x/N}$$

$$F(\omega) = \frac{1}{N} \cdot \sum_{x=0}^{N-1} f(x) \cdot e^{-2\pi j\omega x/N}$$

Wichtige Funktionen



Dirac-Impuls



Kamm-Funktion

Dazugehörige Spektren

Impuls-Funktion: Spektrum $\equiv 1$

- ◆ alle Frequenzen enthalten

Kamm-Funktion: Spektrum = Kamm!

- ◆ Je weiter der Kamm im Ortsraum,
- ◆ desto enger der Kamm im Frequ.-Raum!

$$\text{comb}_T(x) \Leftrightarrow \text{comb}_{1/T}(\omega)$$

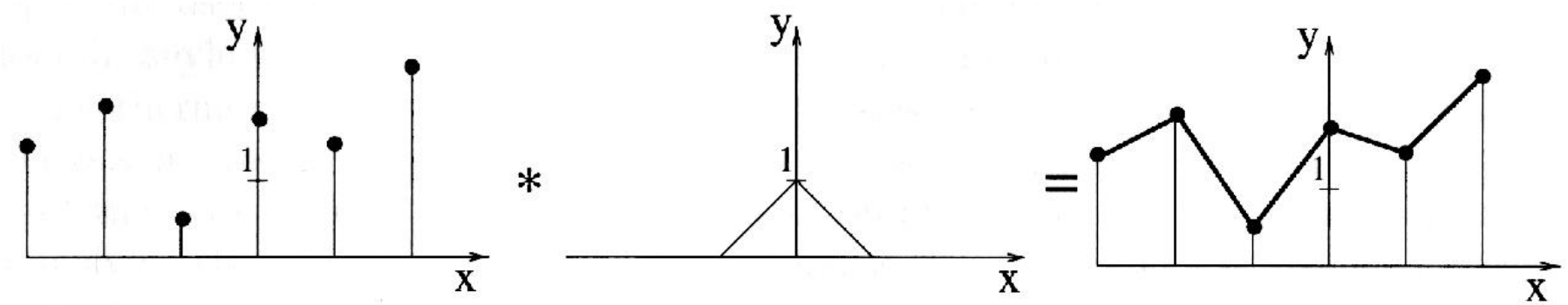
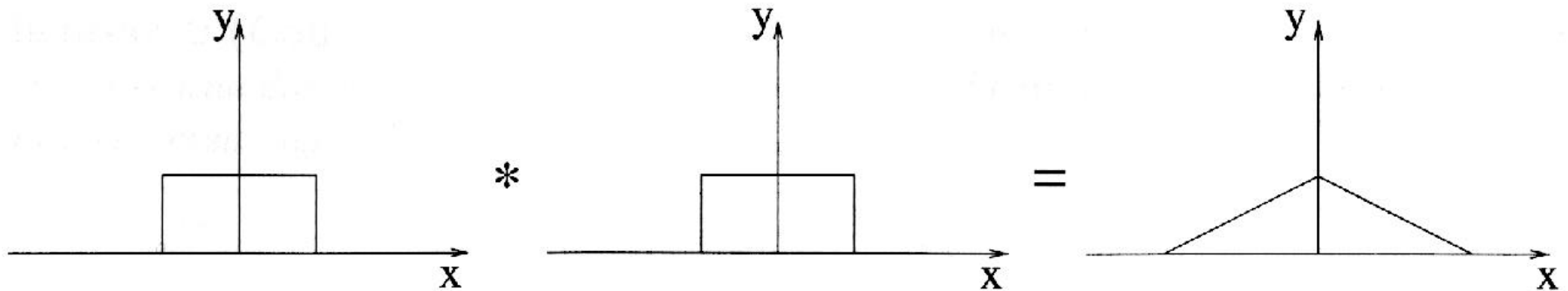
Wichtige Operation: Faltung

Faltung:

- ◆ $h = f \circ g$
- ◆ Input-Funktion f , Filter-Funktion g
- ◆ $h(x) =$ gewichtetes Mittel
von $f(t)$, $t \in [x-w/2, x+w/2]$

$$h(x) = (f \circ g)|_x = \int_{t=-\infty}^{\infty} f(t) \cdot g(x - t) dt$$

Faltung – Beispiele



Faltung und Multiplikation

Ortsraum

Frequenzraum

Faltung

\Leftrightarrow

Multiplikation

Multiplikation

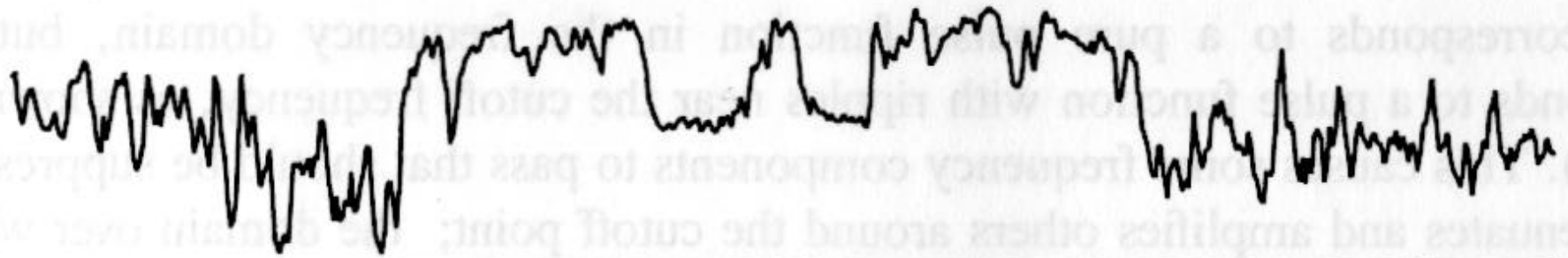
\Leftrightarrow

Faltung

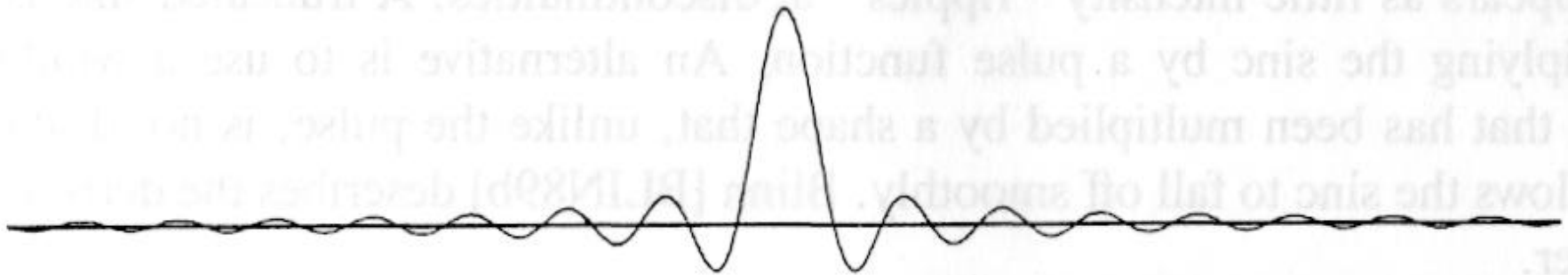
$$f \circ g = F \cdot G$$

$$f \cdot g = F \circ G$$

Low-Pass Prefiltering

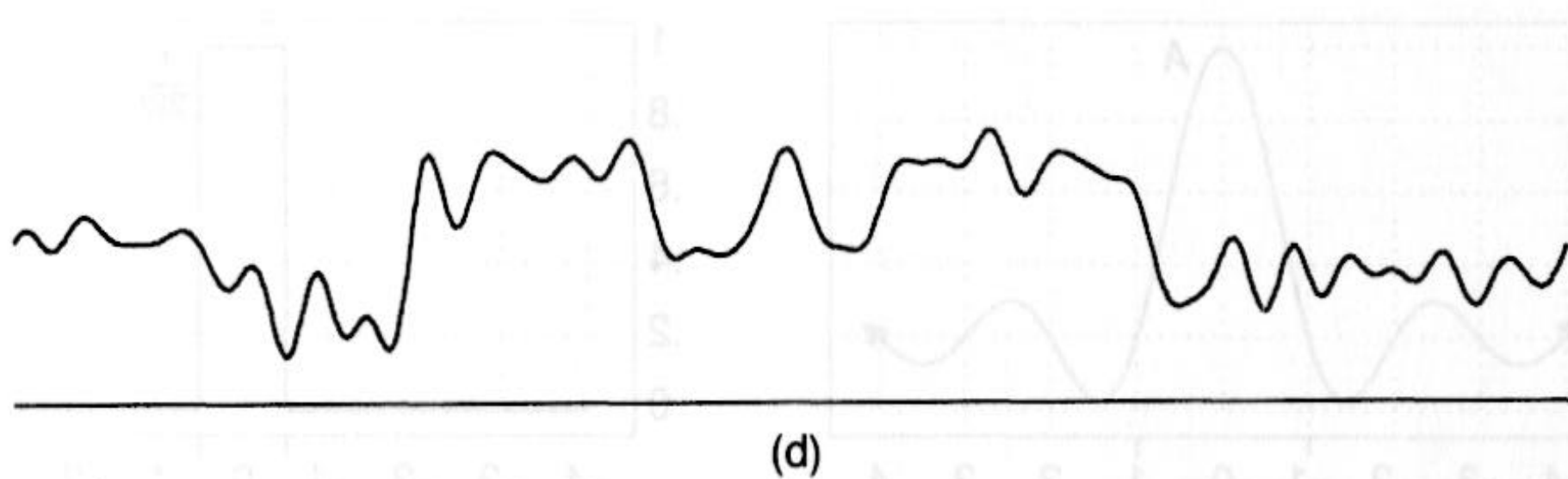
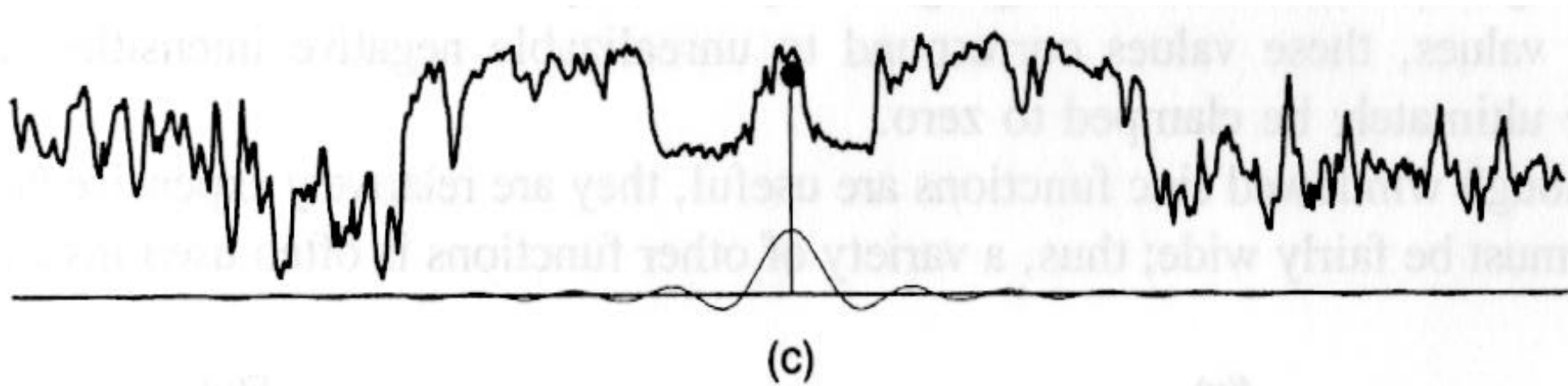


(a)

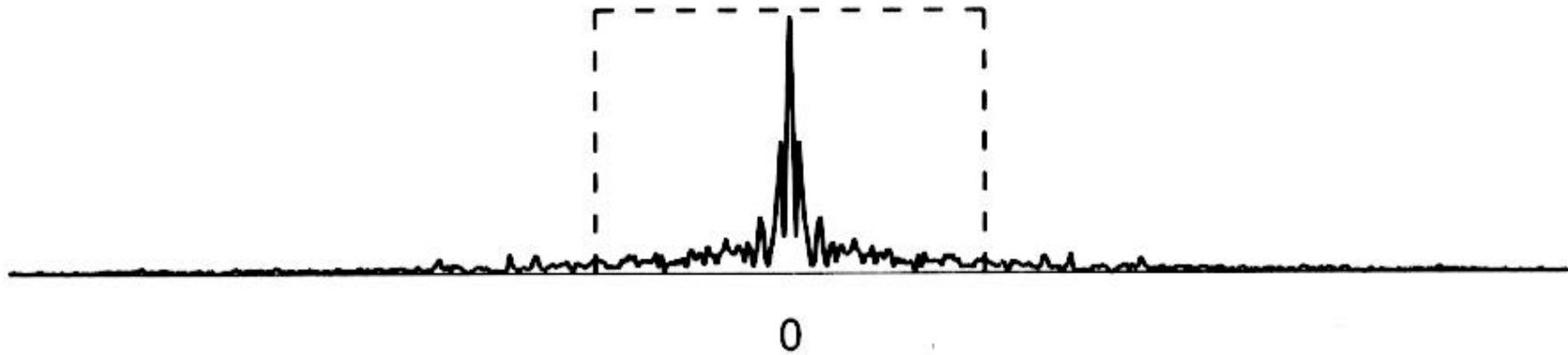


(b)

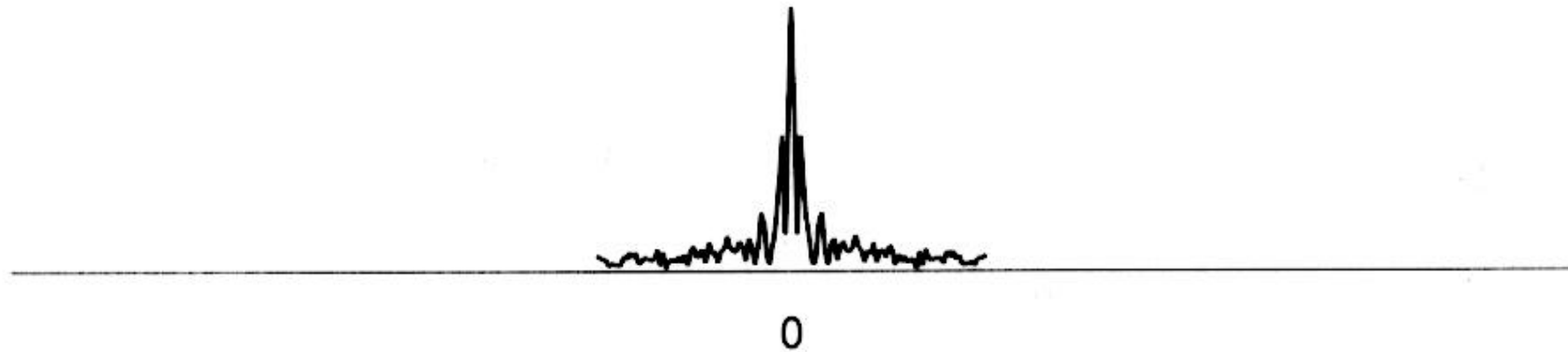
Low-Pass: Faltung mit sinc



Low-Pass im Frequenzraum



(c)



(d)

Sampling

Sampling:

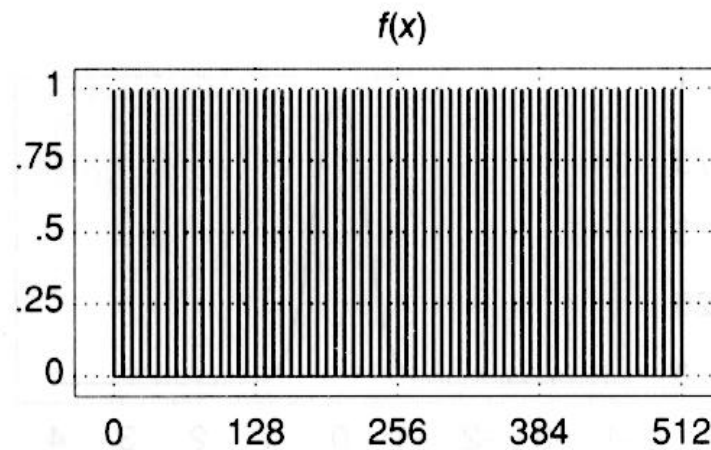
- ◆ Repräsentation durch Beispiele
- ◆ Uniform sampling: Beispiele (samples) regelmäßig organisiert (Gitter)
- ◆ Multiplikation mit Kamm-Funktion (Ort)

$$h(x) = f(x) \cdot \text{comb}_T(x)$$

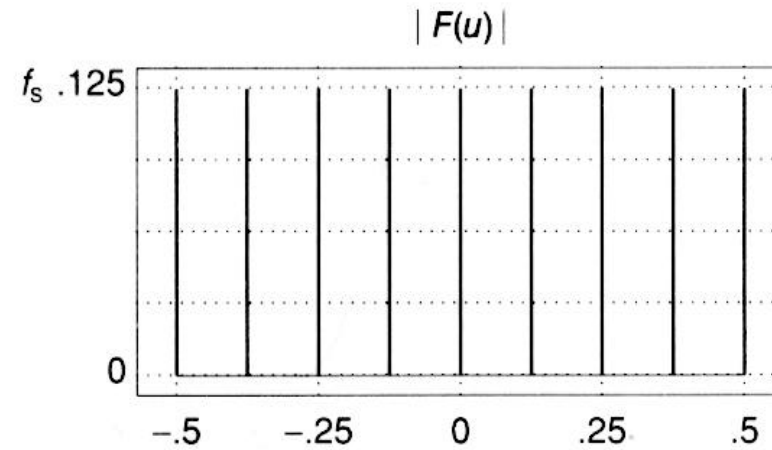
- ◆ Faltung mit Kamm (Frequenzraum)

$$H(\omega) = F(\omega) \circ \text{Comb}_{\frac{1}{T}}(\omega)$$

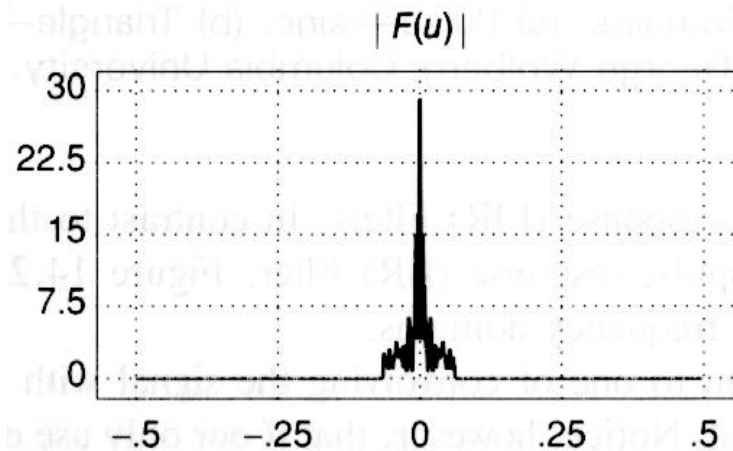
Spektrum diskreter Funktionen



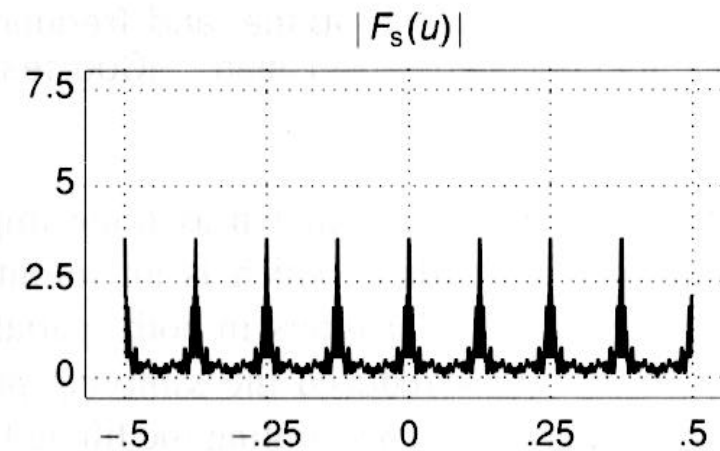
(a)



(b)



(c)



(d)

Sampling Theorem

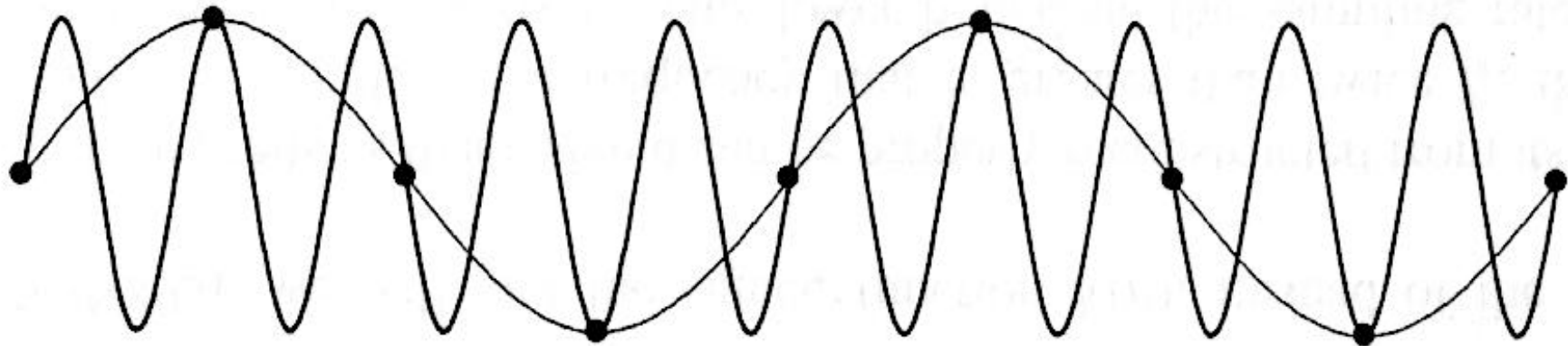
A function f which is

- ◆ band-limited and
- ◆ sampled above the Nyquist frequency

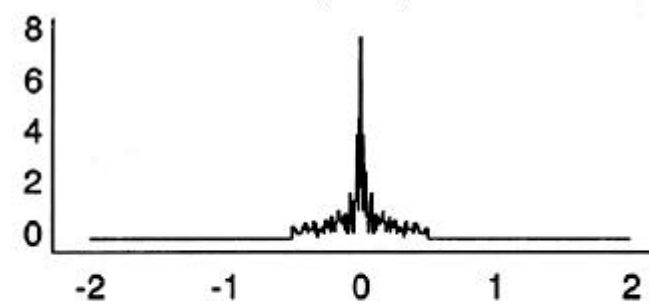
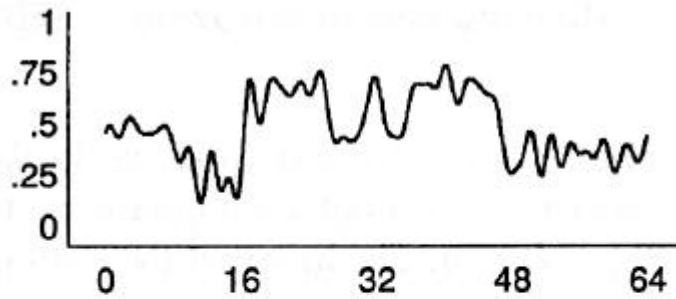
is completely determined by its samples.

Sampling / Nyquist

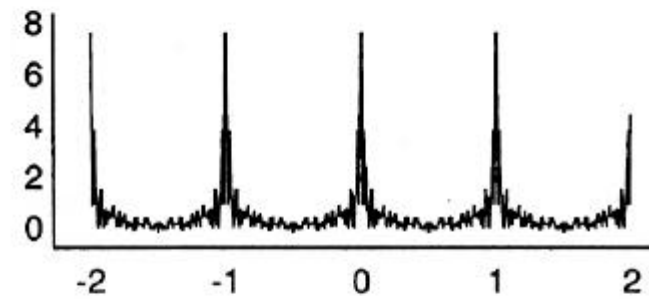
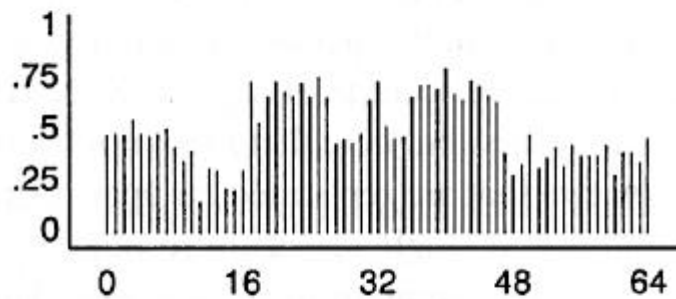
Sampling-Rate zu niedrig \rightarrow aliasing



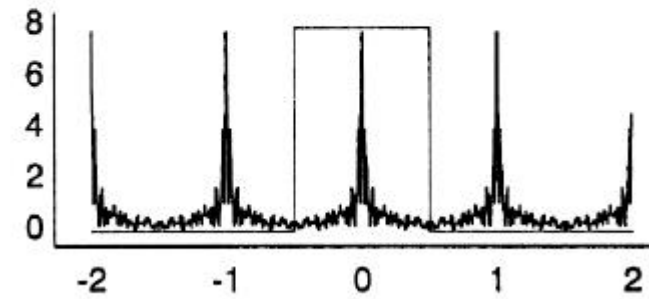
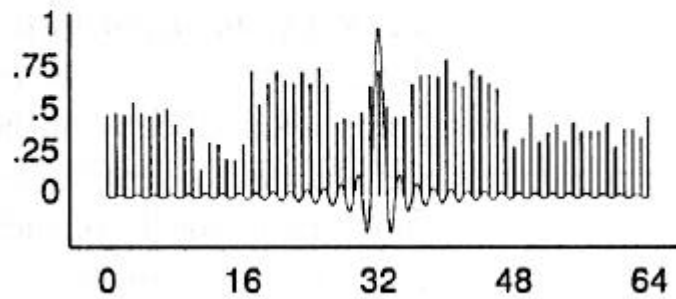
Passende Sampling-Rate



(a)

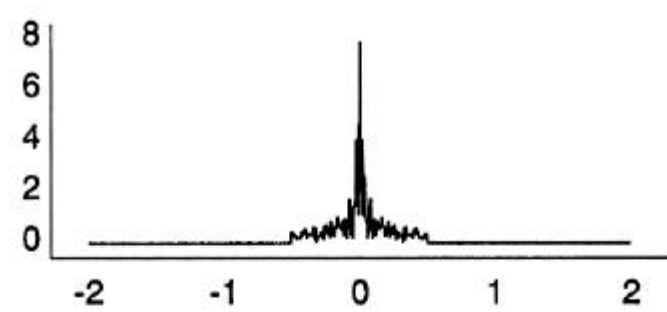
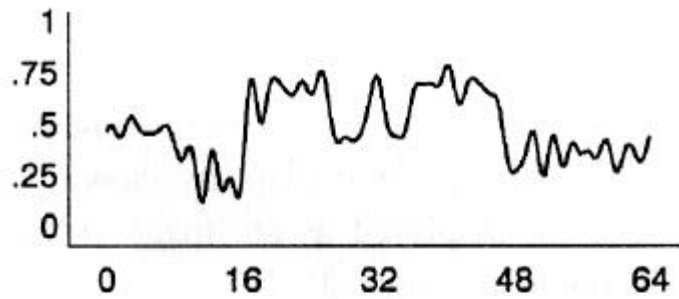


(b)

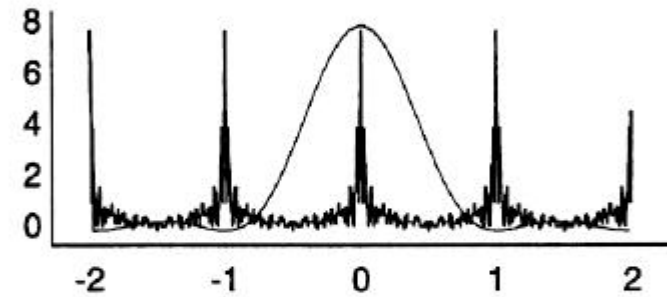
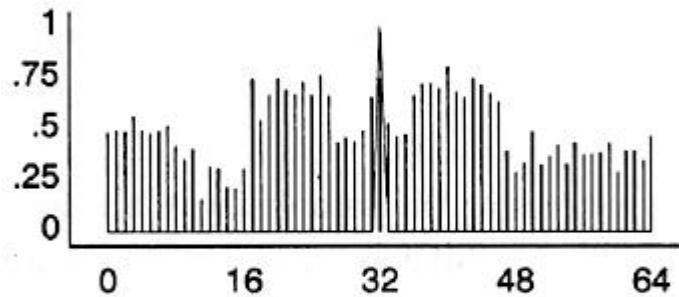


(c)

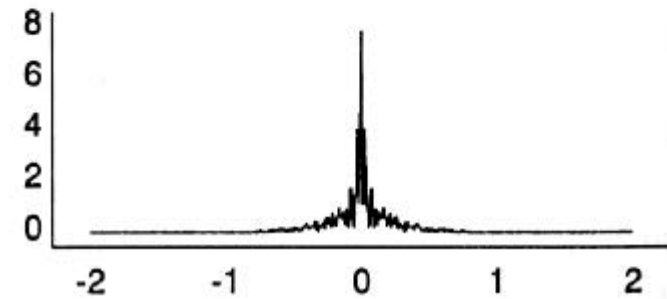
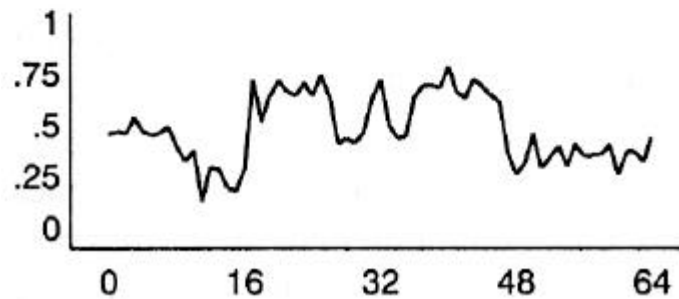
Passende Sampling-Rate (2)



(d)

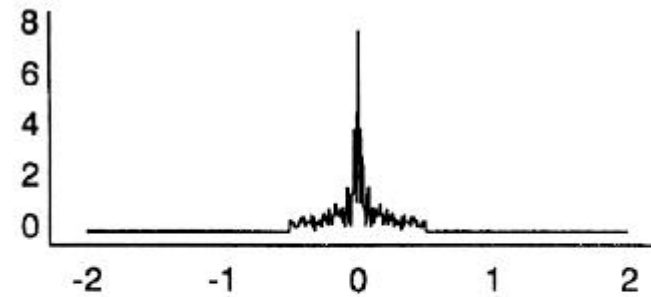
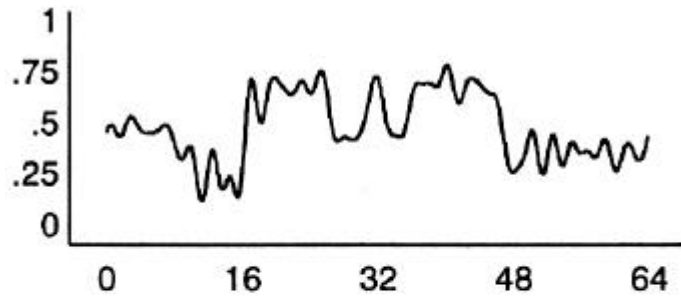


(e)

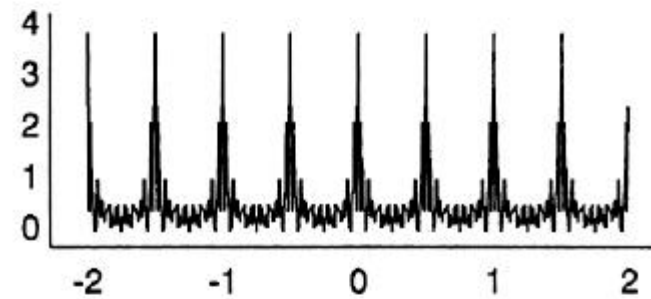
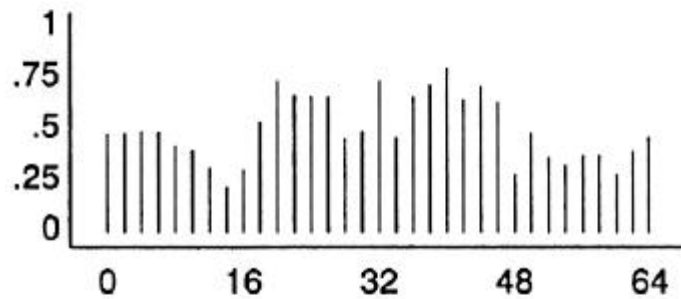


(f)

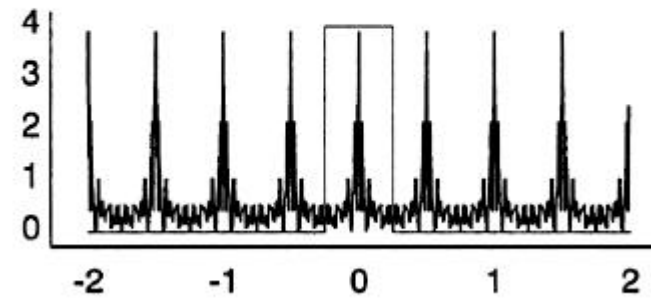
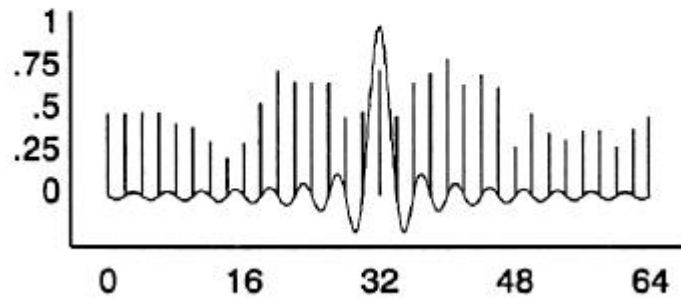
Unpassende Sampling-Rate



(a)

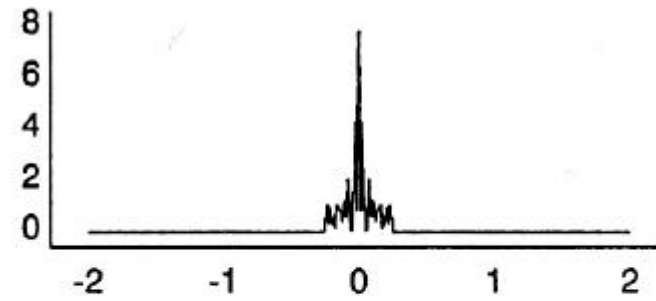
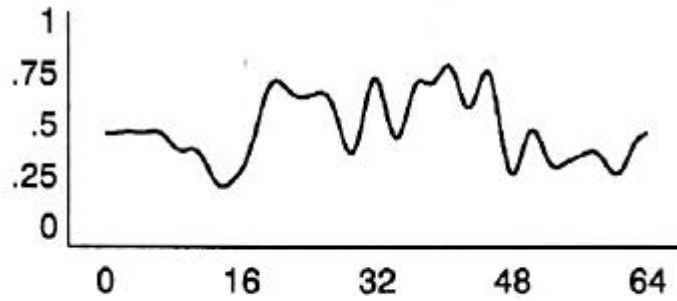


(b)

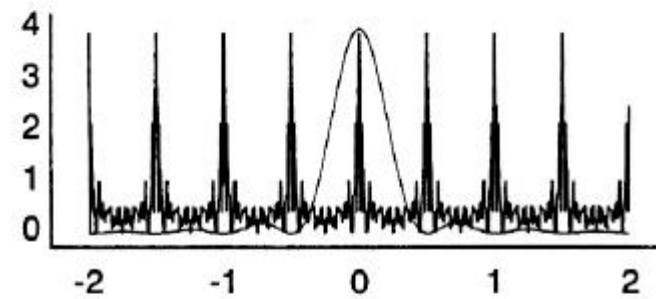
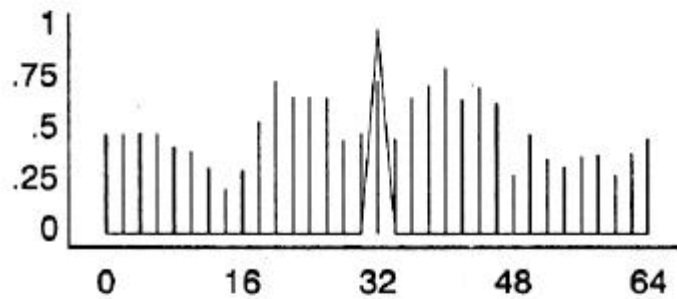


(c)

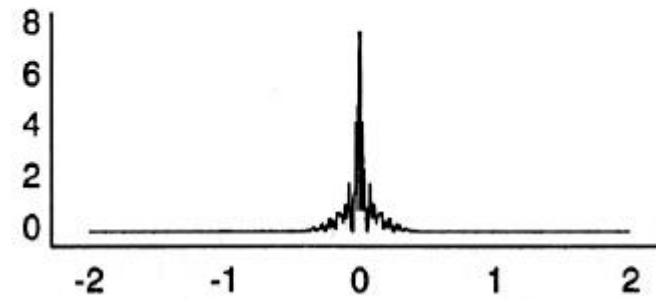
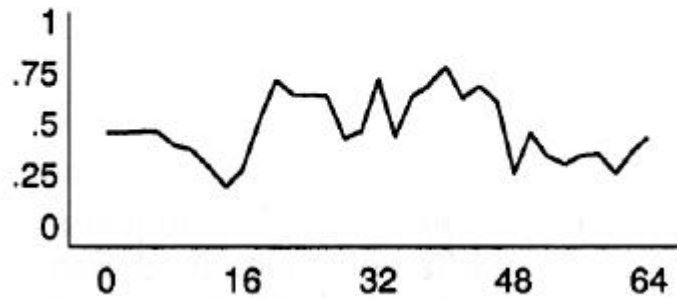
Unpassende Sampling-Rate



(d)

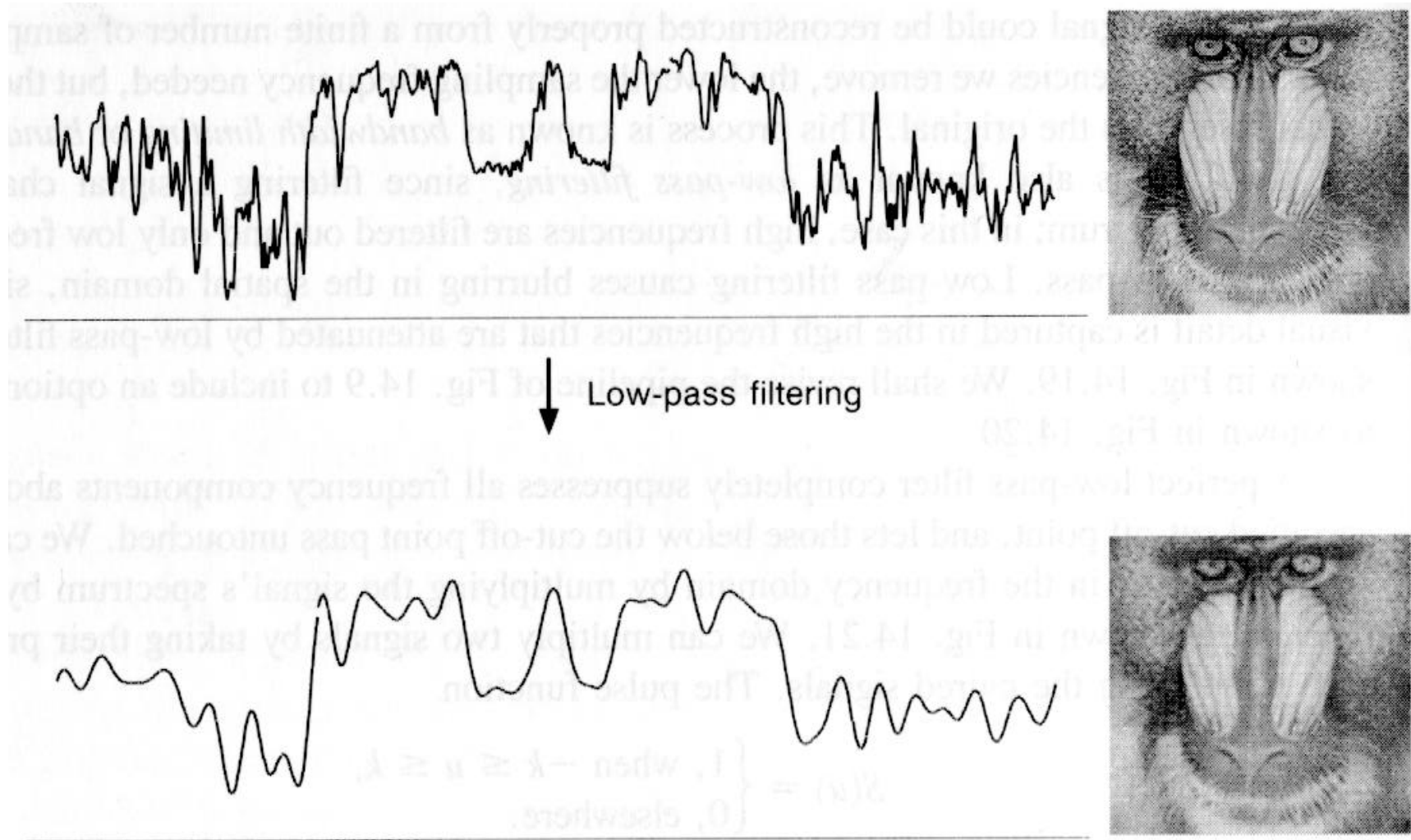


(e)

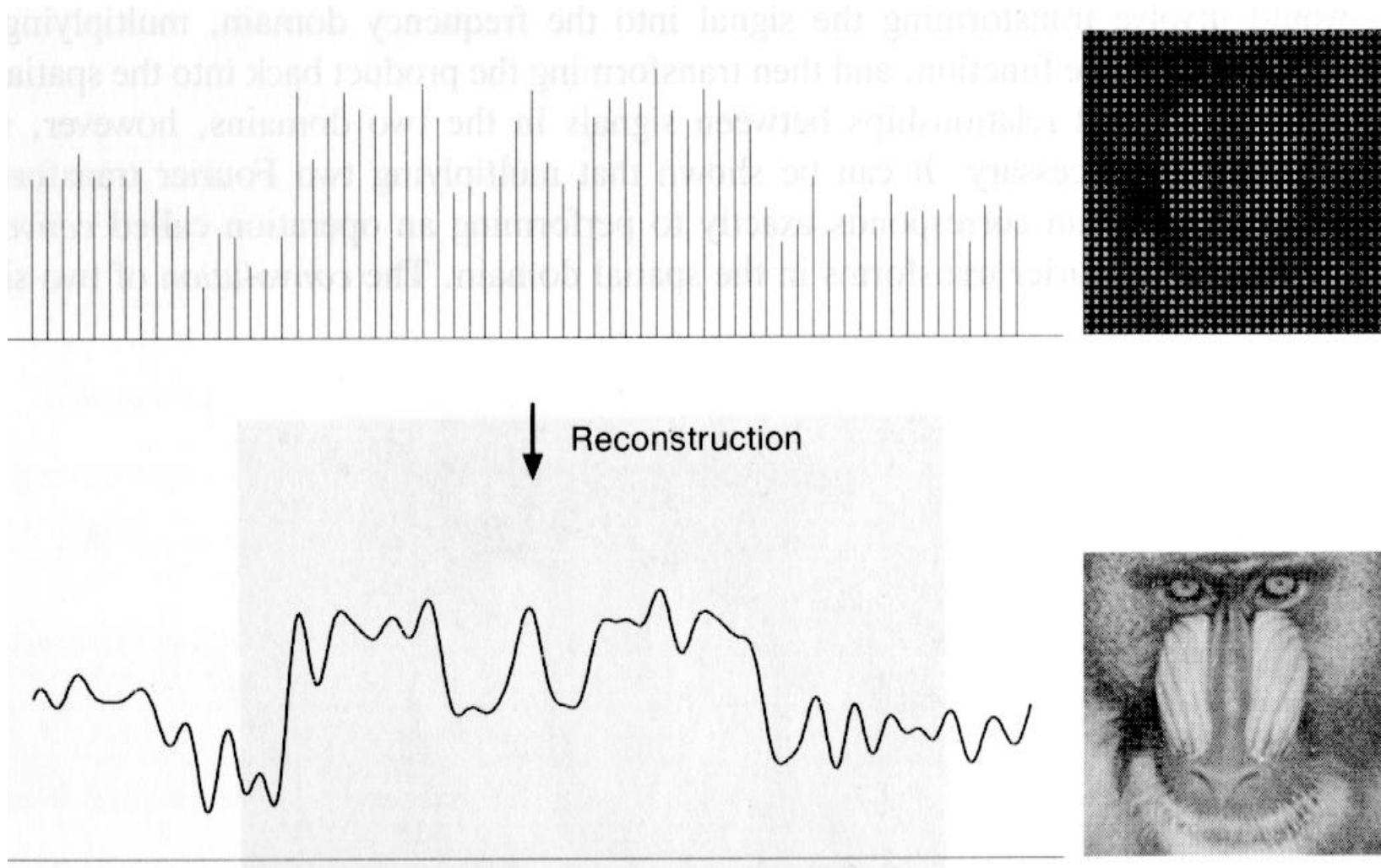


(f)

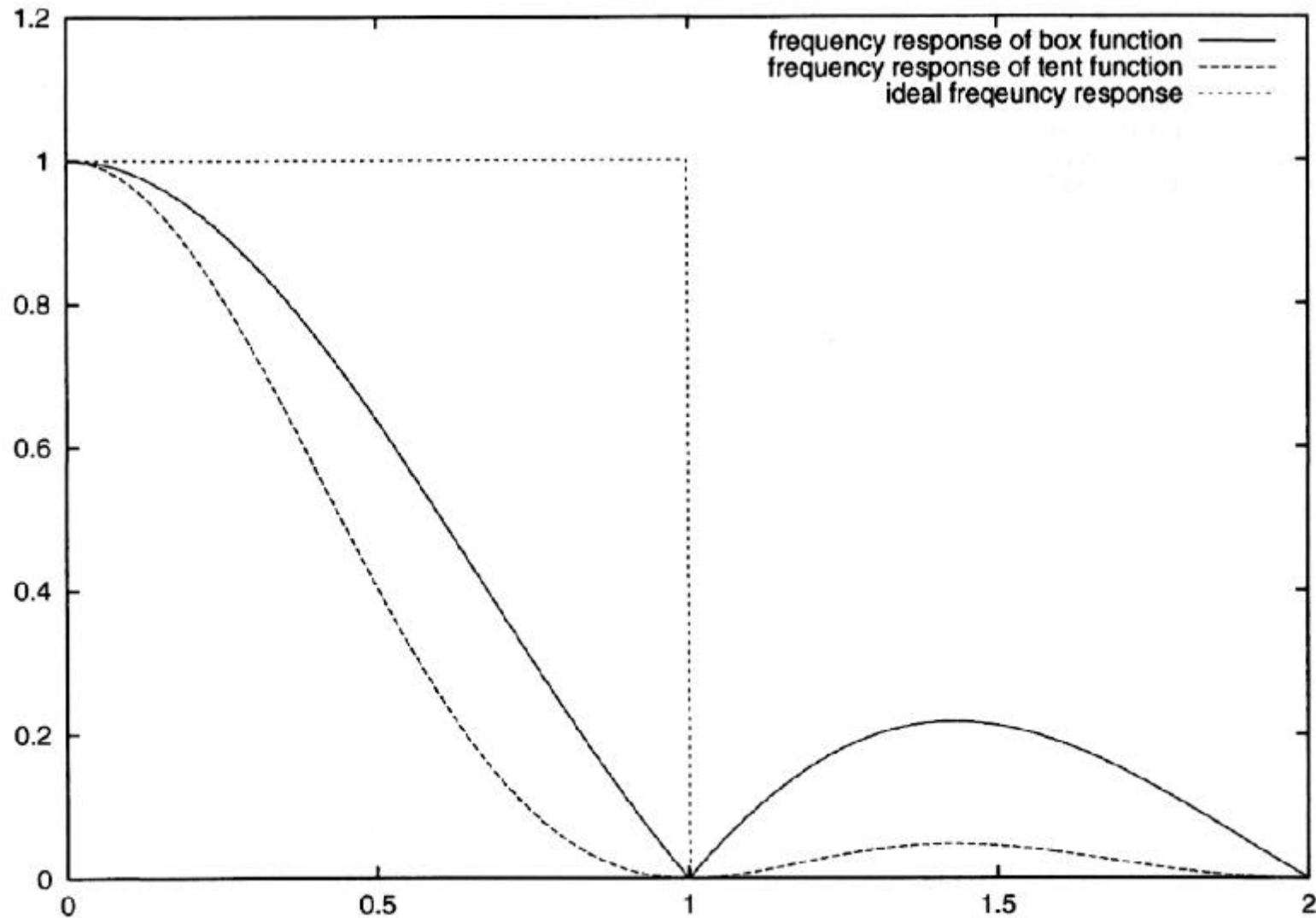
Prefiltering



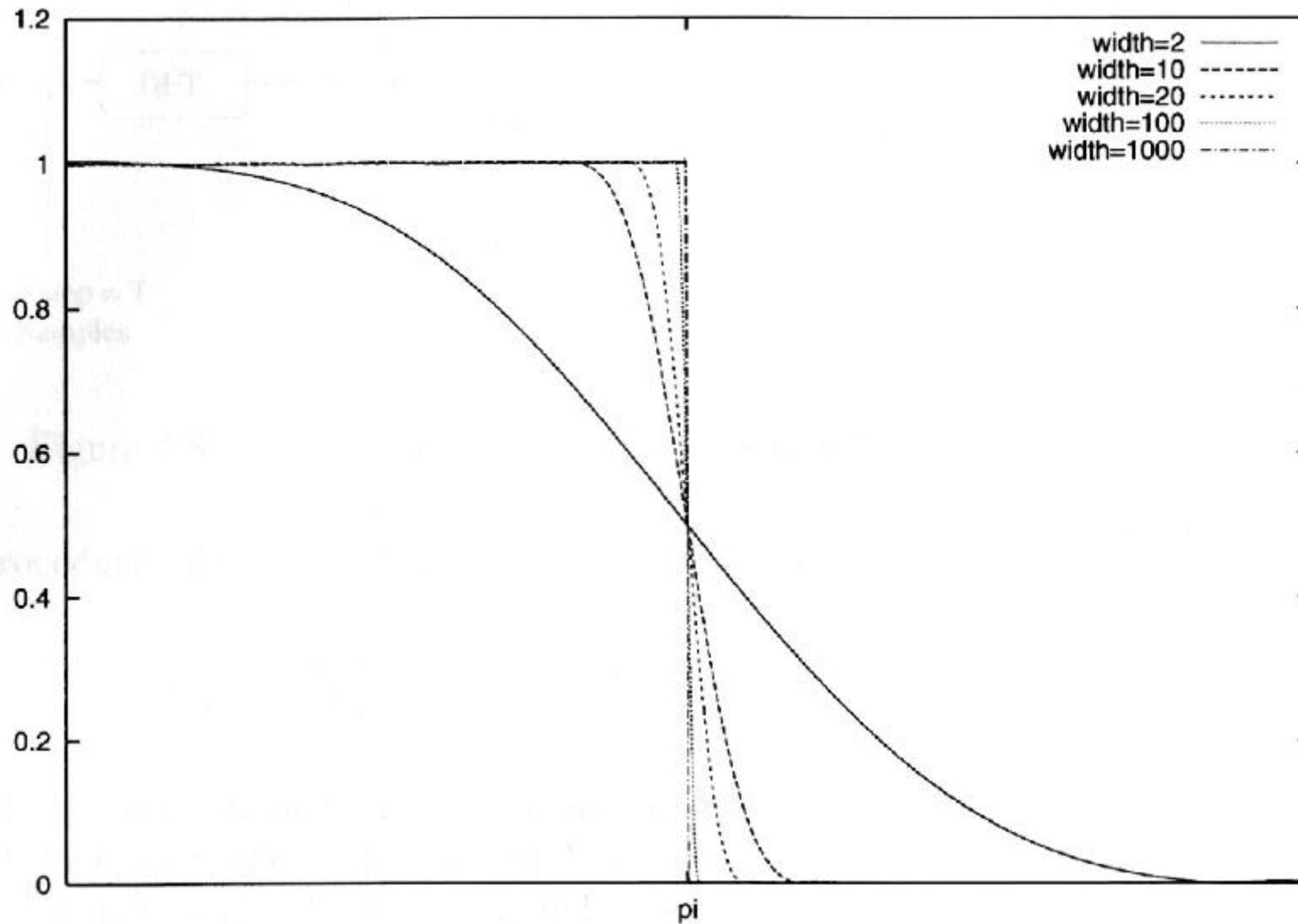
Rekonstruktion



Spektren von box, tent



Gefensterter sinc – Spektren



Aliasing – zwei Arten

Aliasing:

- ◆ hohe Frequ. mischen sich mit niedrigen
- ◆ doppelter Verlust:
 - ◆ hohe Frequ. weg
 - ◆ niedrige Frequ. falsch

Rekonstruktionsfehler

- ◆ Filter \neq sinc
- ◆ Smoothing, ringing

Anti-Aliasing

Prefiltering

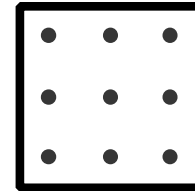
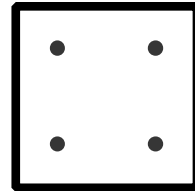
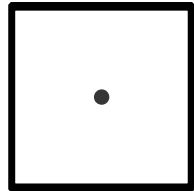
- ◆ Signal **vor sampling** durch low-pass!
- ◆ Ergebnis (band-limited) kann korrekt abgetastet werden (Nyquist-Frequenz)

Supersampling

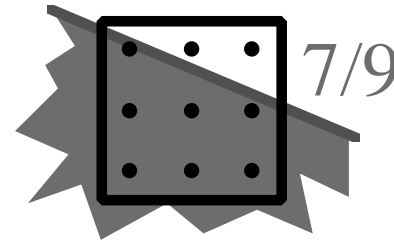
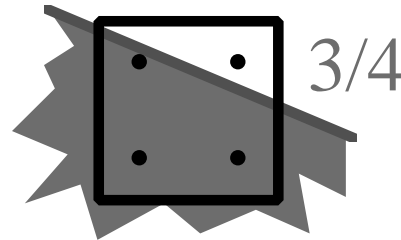
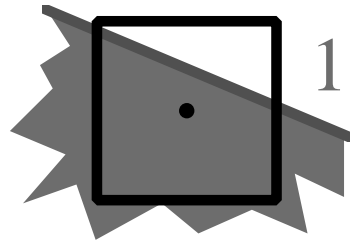
- ◆ Erhöhen der sampling-Rate

Supersampling

Sampling mit erhöhter Frequenz

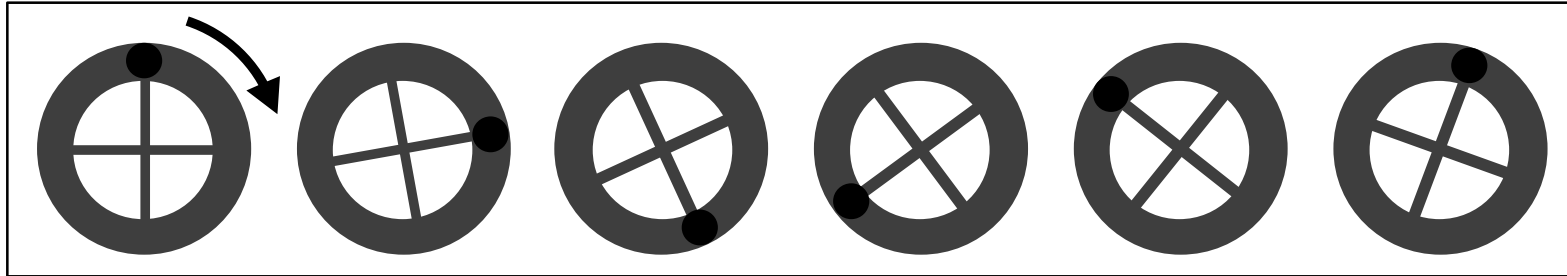


Rekonstruktion mit Tiefpaß-Filter



Aliasing im Zeitbereich

Umkehrung der Drehrichtung



Worming

